DEFENCE R&D ORGANISATION
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Guided Weapons System Design

R Balakrishnan

GUIDED WEAPONS SYSTEM DESIGN

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Dedicated
to the memory of

R Ramaswami Ayyar

my father
without whose help I would not have had
a chance to learn what I have
and
as a fitting commemoration of
the centenary year of his birth
on
11 April 1897

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Cover photo: Line of Sight Guidance Scheme

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FOREWORD

Dr. R. Balakrishnan has brought out his vast experience in design and development of various missile systems in the form of a monograph for missile design. It is a well-prepared document and tremendous efforts have gone in detailing system design with illustrative examples for surface-to-air missile system.

The monograph outlines system analysis and design in nine chapters. The design cycle covering the activity, responsibility and constraints, described in Chapter 1, discusses the existence of diverse configurations meeting the same mission requirements. A detailed methodology for guided weapons system study based on essential functional features is developed in Chapter 2. The four important elements of the design problem are highlighted here and an admissible solution region is obtained for a fighter aircraft and a sailplane as case studies. In Chapter 3, a system model is described to include observation device, navigation and control system, and warhead effectiveness through miss distance, for a given target trajectory. Mathematical models based on command guidance policy have been developed in Chapter 4 and various important parameters like gain (k), velocity ratio (V_m / V_t), command update time and rise time are evaluated for a surface-to-air weapon system. Similar mathematical models are developed in Chapter 5 for a chosen proportional navigation guidance policy. In Chapters 6, 7 and 8, the specification for aerodynamic configuration design, propulsion system design and structural design of the configuration have been generated using system model studies and nomograms to obtain admissible solution range for different characteristics of guided weapon systems have been generated. The levels of interaction in a guided weapons system design are discussed in Chapter 9.

This monograph, indeed, provides an effective approach to a guided weapon system design to meet the users' requirement from the fundamental system studies taking performance as a consideration within the given constraints. This monograph will be useful as a technical reference document and is highly suitable for the young scientists who are entering into the field of design of missile systems.

(APJ Abdul Kalam)
Scientific Adviser to Raksha Mantri

PREFACE

The genesis of this book has a long history. The story starts around December 1963, when the author was posted to the *Fourth Special Weapon's Course* at the *Institute of Armament Technology, Pune*. This was a course of studies on the design of guided weapons. One of the aspects of guided weapons that fascinated the author was the variety of configuration shapes and sizes that he noticed in every issue of *Jane's Weapons Systems*. How were these configuration shapes arrived at? What is the logic behind the choice of their shape? What were the design considerations for determining their shapes and sizes?

This fascination continued even after the author's transfer in 1965 to the Defence Research and Development Laboratory (DRDL) at Hyderabad. Those were the early days of development of guided weapons in India. Very little data for the design of guided weapons was available since most of the information pertaining to guided weapon systems was classified. The only set of declassified books pertaining to guided weapons that was freely available then, was the *Merril Series* of books on guided weapons. Naturally, this set of books formed the early source of information. However, as these books were based on the knowledge gained from the development of guided weapons during the Second World War, these books did not contain information on the later developments in this field. While, there were detailed discussions on the guidance policies and on the preliminary design of systems, they did not provide the logic for the choice of the configuration shape and size. The quest for finding answers to these questions therefore became the mission of the author, throughout his stay at DRDL.

The initial efforts at DRDL, during the first decade of its existence, were directed towards the design and development of a first generation, man-portable, wire-guided, anti-tank missile. Even though the system was a seemingly simple one, its design taught valuable lessons on the need for systems approach to weapon design. *This was a radical departure from the practices prevailing then.*

In the second decade of its establishment, DRDL set out to develop a long-range surface-to-air weapon system. Under this project, initially, the efforts were directed towards indigenous development of a foreign design. The technology of the electronics subsystems used in this foreign design was, however, no more current. The efforts involved in redesigning these subsystems using the state-of-art electronic devices was found to involve as high an effort as was needed to take on a new design altogether. It was soon realised, therefore, that this route was not worth pursuing.

At this stage, Government of India thought it essential to ascertain the capability of DRDL to take on *ab-initio* development of guided weapons. For this assessment, a committee headed by the late Dr Brahma Prakash visited the Laboratory. They had detailed interactions with scientists engaged in the development of each of the specialised subsystems of the guided weapons system.

During this period, the author was leading the Aerodynamics Division of the Aeronautics Group, which was then headed by Mr AV Ranga Rao, an eminent senior scientist. As part of the effort needed to prove our capabilities to take on *ab-initio* design, the systems study methodology for the design of the aerodynamic configuration for guided weapons was developed. This development was published in an internal report—*DRDL 031 100 6002 dated February 1975* — authored by AV Ranga Rao, R Balakrishnan and RN Agarwal. This report on *Aerodynamic Design* was the first serious attempt at DRDL for applying the systems study method to missile configuration design.

Soon thereafter, the author was made a specialist member of a study team formed at the Armament Research & Development Establishment (ARDE), Pune (one of the sister establishments under the DRDO) for the development of a *Kinetic Energy* gun-launched ammunition. As a member of this team, the author applied the systems study methodology for the design of the aerodynamic configuration for the *FSAPDS* ammunition. This approach enabled the author to identify the configuration characteristics that were essential for its design and to determine the admissible range in their values.

After the successful completion of this assignment, the author was nominated as a member of yet another study team at DRDL for the design of a low altitude, quick reaction time, short-range, surface-to-air guided weapon. The *French CROTALE* and the *British RAPIER* systems are two well-known representatives of this class of weapon.

As a member of this team, the author extended the systems study methodology to identify the essential *configuration characteristics* that were needed for its design, to meet the user's performance expectations in the weapon. The author was in for a pleasant surprise when he identified in the solution region thus found, the configuration characteristics of the CROTALE and the RAPIER systems also. The surprise was all the greater when one realised that CROTALE and RAPIER followed two different control philosophies. While CROTALE employed canard controls, the RAPIER system used a tail-controlled configuration. The systems study method was thus able to identify all the configuration solutions to the design problem. *The methodology could thus provide the key to the logic for the choice of the shape and size for the configuration.* The successes of the systems study methodology was thus an exhilarating experience for the author.

This finding formed subsequently the core of the author's doctoral thesis on the *Aerodynamic Configuration Design for Cruciform Missiles*. The Indian Institute of Technology at Madras, under their external registration scheme, in 1985 conferred on the author the *Doctorate* degree in *Aeronautical Engineering*, based on this thesis.

Before his retirement from DRDL in December 1993, the author completed yet another system study exercise on the design of a *shoulder fired missile*, aimed against low-flying targets. This study on the shoulder fired system was based on the application of the *proportional navigation policy* to guided weapon design. The earlier systems study exercise undertaken by the author to counter a low altitude threat was based on the application of the *Command Guided Policy* to missile design.

The successes of the systems study methodology for the design of guided weapon systems following two different guidance policies, indicated that the systems study method was the key to missile design. Further, it was soon realised that the applicability of the systems study method was not exclusive to guided weapon design alone but was applicable to almost all engineering designs. The author therefore felt that the utility of the systems study methodology to solve engineering design problems should be given wider publicity. With this in view, the author decided to engage himself in propagating this methodology, as part of his post-retirement plans.

At this juncture, three events occurred, almost in quick succession. The first was a request to the author from *DRDO*, in August 1994, to write a *Monograph on Guided Weapon Systems Design*. The second was an invitation in October 1994, from the Aerospace Department of the *Indian Institute of Science, Bangalore*, for delivering a series of six lectures on guided weapon design. There was yet another invitation from the Aerospace department of the *Indian Institute of Technology, Bombay*, to deliver a similar series of lectures on guided weapon design in February 1995. The author readily accepted all the three assignments, since the effort involved in these were complementing each other. He felt that the preparation for a lecture would be a welcome pre-requisite before one embarks on writing on the subject.

The author, therefore, set out to prepare the course material needed for the first series of lectures at the Indian Institute of Science. He then updated this material based on the feedback he received from the staff and the students of IISc. The revised notes formed the course material for the next series of lectures at the Indian Institute of Technology, Bombay. With further feedback from the staff and students of IIT, he revised the notes once again, and these formed the basis for this monograph. Subsequently, on completion of the first draft of the monograph, at the author's request, *Lt. General (Dr.) VJ Sundaram, Director, DRDL,* organised a review of the monograph by a team of scientists from DRDL. The review was chaired by *Mr Ranga Rao,* who is one of the Indian pioneers in the field of guided weapon design. The suggestions offered by the review team have been incorporated in the final monograph.

This monograph is, therefore, a record of almost all the efforts put in by the author in his quest to find the logic for the shape and size of the configurations for guided weapons.

The author would like to thank all the scientists with whom he had interacted at DRDL during his quest for the answer to the configuration design problem. In particular, the author would like to thank *Mr K Anandha Narayanan,* a young scientist of DRDL who developed the computer simulation programmes which are needed for validating the results presented in this monograph. The author would also like to thank all the students and staff of IISc Bangalore and IIT, Bombay, who helped him finalise the sequence of presentation. Last but by no mean the least, he wishes to thank *Lt. General Sundaram,* for organising the review of the draft monograph before it could be finalised. The author would like to thank, in particular, *Mr Ranga Rao* and the review team for the valuable comments they offered during the review.

The author wishes to thank *Dr APJ Abdul Kalam* who, as the head of the DRDO, encouraged him to write this monograph on systems study of guided weapons. The author would also like to thank him for writing the foreword for this monograph. He would like to thank *Director, DESIDOC* also, through whose office, the necessary support for this effort was canalised by DRDO.

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CHAPTER 1

INTRODUCTION

One finds an amazing variety of missile configuration shapes listed in every issue of Janes Weapon System. While some are delta-wing configurations, others are either trapezoidal-wing configurations or rectangular-wing configurations. Some have cylindrical bodies with hemispherical noses, while others have either ogival or conical forebodies. Likewise, one finds variations in the type of control systems employed—canard-controlled configurations and tail-controlled configurations—while a few are just thrust vector-controlled configurations. Even in missiles of the same class, one finds variations both in their external shape and in the type of control system employed. Consider, for example, the class of low-altitude surface-to-air missile systems represented by the French CROTALE, the British RAPIER and the Euro-Missile Consortium's ROLAND. Where CROTALE uses canard controls, RAPIER employs tail controls and ROLAND is a thrust vector- controlled configuration. The configurations of these missiles have been drawn to the same diameter as reference, though their diameters differ in magnitude (Fig. 1.1). Seeing such a diversity in their configuration (though these are designed to meet an almost identical mission requirement), one wonders on what basis these configuration shapes have been arrived at. Surely, these have not been stumbled upon by chance! Is there a logic for arriving at these diverse configuration shapes to meet an identical mission requirement?

Missile design is a time-consuming and expensive venture. The user is also a very demanding customer who would like to be assured at every stage of its design, that the design was based on sound logic. There was, therefore, a very high probability of the design meeting his requirements. Considering such a premise, it is reasonable to presume that every configuration variation listed in Jane's must have been evolved after considerable deliberation and could not have been arrived at by chance. By the same token, it is logical to expect that there can be many other configuration solutions also that would have met the user's expectations equally well. If so, how can one identify these solutions and what are the reasons for discarding them in preference to those that are listed in Jane's?

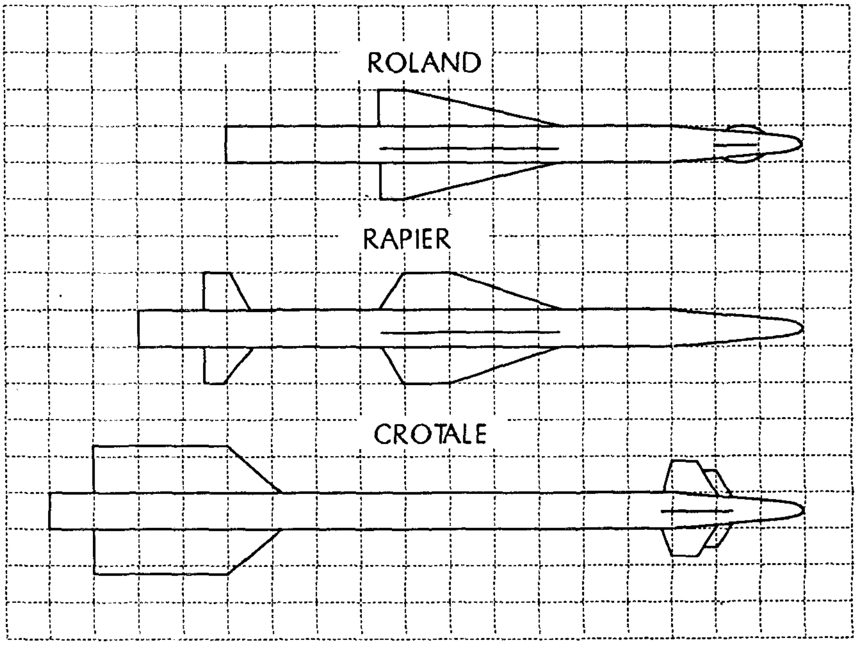

Figure 1-1. Comparison of CROTALE, RAPIER and ROLAND

Some of these questions are proposed to be answered in the subsequent chapters.

The author has not come across any reference in the published literature dealing with the methods for missile configuration design. At the same time, it is improbable that a vacuum exists in this area. Perhaps, the information on missile design is classified and, therefore, its availability could be severely restricted. What is presented in the following chapters, therefore, is a method developed by the author[1] after many years of patient search for the Rosetta Stone for unlocking the secret of missile design. Having developed the method, in hindsight, it appears that the method is not something peculiar to missile design alone, but is universally applicable to all engineering designs. Therefore, there is no need to be secretive about it at all, in the first place.

1.1 ENGINEERING DESIGN

According to Dixon[2], the objective of engineering design is to help one develop skill in applying what one has learnt in science and other engineering courses, for finding solutions to practical engineering problems. The objective of the courses in engineering design, therefore, is to help use purposefully and effectively what one has already learnt in the various branches of engineering.

French[3] defines *design* to mean "all the process of conception, invention, visualisation, calculation, and specification of details that determines the form of an engineering product." Thus, design would imply a series of structured activities that are planned for achieving the objective of the creation of an engineering product.

No new engineering product can be created, unless there is a *need* for it. The need is usually expressed as an *expectation* by society, the user or the market in a product. Thus, all activities associated with design start with the enunciation of the requirements by the user, society, or the market. Finding a solution to meet the stated need is 'engineering design.'

Dixon defines *engineering design* as follows:

"Device, *subject to certain problem solving constraints*, a component or a process or a system to accomplish a specified need, optimally, *subject to certain solution constraints*."

As per this definition, the designer is called upon to use his ingenuity to *devise* solutions to the *specified need* of the society, user or the market *subject to constraints*. This phrase occurs twice in the definition, only to emphasise the fact that the designer does not enjoy unfettered freedom in achieving his objective of finding engineering solutions to the specified need. He is constrained both by certain *problem solving constraints* and by *certain solution constraints*. While problem solving constraints refer to the technical limitations the designer may have in finding solutions to a stated need, solution constraints refer to the technological difficulties in realising these solutions.

1.1.1 Problem Solving Constraints

Modern day designs are multi-disciplinary activities. It is quite likely that the designer may not be familiar with all the associated fields of specialisation needed for its design. This limitation would therefore present one of the problem solving constraint. However, to a large extent, this limitation can be overcome by forming specialist design teams comprising designers who have knowledge in the required fields of specialisation.

Again, it is quite possible that the resources needed for finding a solution to the problem may not always be available to the designer. A typical example could be the non-availability of modern supercomputing facility needed for solving the fluid dynamic equations for determining the flow field around flying configurations. All these and many more are the real life limitations that designers face in their day-to-day activities. *The designer is enjoined to find a* *solution to the specified need, in spite of these problem solving constraints.*

1.1.2 Solution Constraints

When the designer succeeds in finding solutions to the stated need, there can still be constraints in translating these designs into a product. These constraints could be due to the lack of resources needed for their realisation as a product. Typically, the non-availability of special machinery, material or the processes needed for their manufacture would represent examples of such solution constraints. *The designer is expected to find a solution to the stated need, overcoming such solution constraints.*

Finally, after the designer succeeds in finding a solution to the stated need, overcoming the various constraints, the designer has to check if the solution so obtained, is *optimal* in some sense. This is the *figure-of-merit* by which a design is rated by society, user or the market. Further, the solution so obtained should not have any deleterious effect on the environment, the surrounding ecology or on the consumption of energy before it finds acceptance by society.

It is thus noticed that the definition of engineering design, as given above by Dixon, takes into consideration all the practical aspects inherent to a design problem, and is therefore, the most comprehensive definition for it. The variations in the configuration details of CROTALE, RAPIER and ROLAND could therefore be attributed to the constraints that the designers might have faced during their designs and to the optimality condition set by their respective users.

All possible solutions to a design problem should be evaluated, both by the designer and the user, for their economics and ease of operation and maintenance before the final selection is made. The final selection is made by the user, society or the market, based on their *acceptability criteria*. We realise now that CROTALE, RAPIER and ROLAND are the design solutions to counter an identical threat from low-flying aircraft, that have passed the acceptability criterion set by their respective users.

The design cycle, as described above, is depicted in Table 1.1. It may be noticed that the arrows between the activities in the design cycle point both ways. This is only to emphasise that each step in the cycle is a two-way activity and a lot of interaction takes place between those who are responsible for these activities. In actual practice, even the sequence of activities depicted herein is not so important, as the necessity of these stages in the design cycle. It is also pertinent to notice that the design cycle starts with the enunciation of the need by the user and ends with the user making the final selection out of the various design options. Thus, the user, market or society is supreme and dictates the progress at every stage in the design cycle. It is therefore essential to carry the user along and develop his trust in the designers by keeping him associated with the designers throughout the design cycle. At times, it may even become necessary to educate the user about the repercussions in pursuing a certain line of solution to a stated problem. Such a rapport between the designer and the user goes a long way in each appreciating the other's viewpoint and is essential for success in any venture.

Table 1-1. The design cycle

Activity	*Responsibility*	*Constraint*
Enunciation of the need	User, society, or market	
⇓⇑		
Finding all possible design solutions to the stated need	Designer	Knowledge and experience in the multi-disciplinary field of engineering
⇓⇑		
Realisation of all possible design solutions	Designer	Availability of special material, machinery and processes
⇓⇑		
Evaluation of different realisable solution options	Designer, user	Economics of production, ease of operation and maintenance
⇓⇑		
Selection of the final solution	User, market or society	Acceptance criteria

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CHAPTER 2

DESIGN PROCESS

2.1 IDENTIFICATION OF DESIGN SOLUTIONS

Every design activity starts with the enunciation of the need by the user. It was brought out in chapter 1, that the very same need admits of more than one solution depending upon the constraints under which the designer seeks the solution. The final choice of the solution is however based on the user's acceptability criterion. How does one start the design process to identify all the solution options to a given design problem? This is the question that is sought to be addressed in this chapter.

Having enuncialed the need, the user does not consern himself with the designing process; as long as the design meets his stated needs. Therefore, *the statement of the user's needs should be the starting point of all design activities.* However, the enunciation of the need is very often neither precise nor described in a language that is meaningful for starting the design process. The designer will therefore have to interact with the user quite often before he can translate the user's *qualitative requirements* into a set of *technical specifications.* These specifications involve *quantitative data* that are so very necessary for starting any meaningful design activity. This is a crucial stage in the design cycle that is known as the *problem definition stage.*

The user's requirement is evolved in one of the following three ways:

- **Modification**—The user may find a shortcoming in an existing product, which he would like to be eliminated in the new design. In such a case, a critical study of the existing product would reveal the *improvements* that can be effected. Technical specifications for incorporating these modifications in the existing design can then be drawn out based on an identification of the improvement possibilities.
- **Adaptation**—The second type of requirement is based on the user's desire for extending the use of an existing product to a new application area that was not foreseen by the original

designer of the product. In such a case also, a critical study of the existing product would reveal the *stretch potential* that is available in the existing design. Technical specifications for the new design can then be drawn out based on such an identification of the stretch potential.

- ***Ab-initio* Design**— The third type of requirement arises when the user's need is so unique that no existing product can either be modified or adapted to meet the new requirement. Two of the well known examples of this type of requirement are the Russian's need for a device to retrieve objects from the moon and the American's need for a vehicle to take a person to the moon and to safely bring the person back to earth. The Russian requirement was met by the successful design of the *Moon Buggy*. The American need resulted in the design of the *Moon Lander* that ferried Niel Armstrong to the moon and brought him safely back to earth. How were the technical specifications for these requirements arrived at? Surely, there were no systems existing then on which the specifications for the new design could be based.

Both *modification* and *adaptation* draw their specifications for the new design by conducting *reverse engineering studies* on an existing design. The methods of reverse engineering studies are well known to be detailed further. In the case of *ab-initio* design, however, where there are no existing designs to base the new design on, one has to seek alternate means for drawing out the specifications for their design. One such method is the method of *systems study*.

The technique of systems study is used for identifying the *essential functional features* that are needed in designs to meet a new requirement specified by the user, for operation even in unknown situations or in a new environment. Once the needed essential system functional features have been identified, there are many different ways of implementing them in specific designs, depending upon the constraints under which the designer is seeking a solution. All these are valid design solutions.

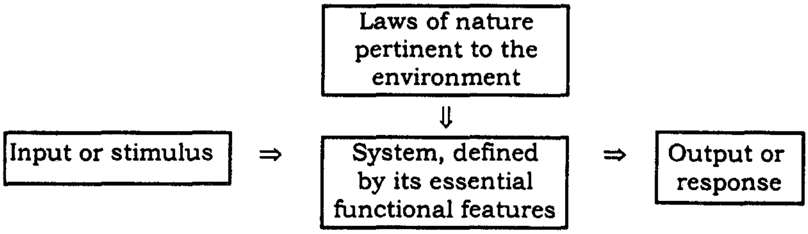
The success of systems study is based on the careful application of *known scientific facts and principles* to find answers to the design problem. It is based on the fact that, in all walks of life, the *response* of a system *to a given stimulus is predictable, if the laws* governing the system in that environment *are known*. Since the laws governing the system are precise, *the response* of the system to every stimulus *is unique and assessable*. It is then a simple matter of tailoring the set of system functional features appropriately, in order to evoke the response from the system that is desired by the user, society or the market. This set of essential system functional features then form the technical specifications for the detailed design of the system. This, in a nut shell, is the principle governing systems study.

Generally, there are four elements to every design problem. These are:

- The system, defined by its functional features,
- The input or stimulus that is imparted to the system,
- The output or the response that is desired by the user from the system, and
- The physical laws governing the system in its operating environment.

These four elements are interdependent. Their interdependency, as given by Dixon[1], is shown in Table 2-1.

Table 2-1. Inter-dependence of design elements



Since these four elements are interdependent, each one of them can be expressed as a function of the other three (Table 2-2).

Table 2-2

Given	*Find*	*Process*
Input, the laws governing the system, and the system, (defined by its functional features)	Output	Engineering analysis (Deduction)
Output, the laws governing the system and the system, (defined by its functional features)	Input	Inverse analysis

Contd..

Given	*Find*	*Process*
Input, output, and the system (defined by its functional features)	Laws governing the system	Science (Induction)
Input, output, and the laws governing the system	System (defined by its essential functional features)	Engineering design

It is noticed that engineering design involves the determination of the system functional features that are essentially needed to translate the input to the system into the desired output from the system. The physical laws governing the functioning of the system under the operating environment provide the link between the input and the output. The systems engineer, therefore, considers the system as a *black box* and attempts to determine its *transfer function* in terms of its essential functional features. Its identification forms the crux of systems study.

There are many different ways for realising this transfer function in specific designs, though the systems transfer function is unique. *Detailed design involves the appropriate selection of these essential functional features to realise this unique transfer function.* Depending upon the constraints under which the designer operates, there are many different ways for implementing this set of essential functional features in the design. All these solutions are however valid design solutions to meet the user's need. This, in brief, is the technique of systems study.

How does the systems designer use this inter-relationship in systems study? At the outset, the systems designer takes a *holistic view* of the entire system and defines a model for the system. This model is defined by a minimum set of *essential functional features* that describe, in an *aggregate sense*, the response of the system to a given stimulus. The designer then conducts simulation studies using this model, till the desired response expected from the system is obtained for specified inputs to the system. This set of essential functional features is a unique set. Every system incorporating these functional features in its design would transform the input given to the system into the output, which the user desires from the system. *This minimum set of the essential functional features forms the technical specification for the detailed design of the system.*

By taking a holistic view of the system, the system designer avoided getting into the details of the system and yet succeeded in identifying all the essential functional features needed for its design.

The system designer has identified the *essence* of the system and has thus succeeded in discerning the *woods from the trees.*

At this juncture, it would be better if we take up an example to fix the ideas of systems study developed thus far. A familiar example pertaining to the design of an aircraft is taken up for illustration, since it is a good teaching practice to go from the *known to the unknown.* The method for arriving at the specification for the design of an aircraft to meet a *desired turn-performance* is taken up for illustration. The user usually specifies the turn-performance desired on an aircraft in either one of two ways, depending upon the application.

Case 1. Design of a Fighter Aircraft

In the design of a fighter aircraft, the user would demand a *rate-of-turn* for the new design that is greater than the rate-of-turn capability of his enemy, to out-manoeuvre it in a dog-fight. Thus, the qualitative requirement for the new design would stipulate:

Rate-of-turn $> \dot{r}_{\max}$ *of the adversary*

Case 2. Design of a Sailplane

In the design of a sailplane, however, since the sailplane should always remain within a thermal, the user would demand that the *radius-of-turn* of the new design should be less than the minimum expected radius of the thermal. Thus, the qualitative requirement for the sailplane would specify:

Radius-of-turn $< R_{\min}$ *of the thermal*

Constraint — Tolerance to Lateral Acceleration

In no case shall the *lateral acceleration* experienced by the passengers and contents of the aircraft or sailplane exceed the value that can be tolerated by them. Thus,

Maximum lateral acceleration $\leq a_{\max}$

These are the two ways in which the user defines the turning performance he expects in the new design. With these requirements as inputs for the system study, the system designer employs the systems study methodology to identify the essential functional features that are needed for the design of the system.

2.2 SYSTEMS DESIGNER'S POINT OF VIEW

As pointed out earlier, the systems designer takes a holistic view of the flying system. From his view point, the designer is not able to discern the elements in the flying aircraft, such as wings, fuselage, elevator, rudder, its powerplant, etc. These details are not needed by the designer to describe the turning performance of the aircraft. The systems designer perceives the aircraft only as a *point-mass* moving in three-dimensional space at a certain speed and executing the specified turn. The relevant laws governing the aircraft in turning flight are the *Newton's Laws of Motion.* As per these laws, the heading of the aircraft changes whenever it executes a turn. Therefore, to effect changes to its heading, force lateral to its heading is required to be impressed on it. Thus, the *stimulus* that needs to be imparted to the point-mass to effect the desired turn performance *is a lateral force.* The systems designer now applies Newton's laws to relate the lateral force stimulus needed to evoke the response in turn performance desired from the new design. This relationship between the stimulus and the response is depicted in Table 2-3.

Table 2-3. Relationship between stimulus and response

Stimulus	*System functional features*		*Response*
Case 1. Fighter design			
Lateral force =	(mass × velocity)	×	Rate-of-turn
Case 2. Sailplane design			
Lateral force =	(mass × velocity^2)	÷	Radius-of-turn
Constraint. Lateral Force=	(mass)	×	Limit on lateral acceleration

The knowledge of the physical laws governing the turning flight of the aircraft has thus enabled the system designer to identify the *essential functional features* of the Black Box.

- In the case of the *fighter aircraft* design, the essential system functional feature is identified as the *product of the mass of the aircraft with its flight speed,* and
- In the case of the *sailplane* design, it is the *product of the mass of the sailplane with the square of its flight speed.*

The next task is to determine the ranges in the values of these essential functional features. The selection of these functional features should be such that under no condition of flight should the lateral acceleration exceed the constraint specified for its design. It is noticed that the constraint specification on the permissible lateral acceleration influences only the selection of the flight velocity of the aircraft and *not the mass of the aircraft.* Thus,

Case 1. Fighter Design

$$(V * \dot{r}) \le a_{\text{max}}$$

Case 2. Sailplane Design

$$\frac{V^2}{R} \le a_{\text{max}}$$

For constant values of the permissible lateral acceleration (a_{max}), the graph connecting the flight velocity of the fighter aircraft and its rate-of-turn is a rectangular hyperbola. Similarly, the graph connecting the square of the flight speed of the sailplane, and the curvature of its flight path, 1/R is also a rectangular hyperbola. The admissible values of the relevant parameters lie in a region bounded between the hyperbolae on the one side and their respective axes on the other side. On this solution region, by marking off the ordinates pertinent to the inequality specification on $\dot{r}_{\text{max}}$ and R_{min} the admissible solution region for the flight speed specific to the problem can be identified.

In the case of fighter design,

$$V \le \frac{a_{\text{max}}}{\dot{r}_{\text{max}}}$$

In the case of sailplane design,

$$V \le \sqrt{(a_{\text{max}} * R_{\text{min}})}$$

It is now required to determine the mass and the size of the aircraft. It is however noticed that the constraint specification on the permissible lateral acceleration has no influence on the choice of the mass. Therefore, requirement for the choice of the mass parameter is to be sought elsewhere.

The lateral force is generated on the aircraft configuration by setting the configuration at the required angle of attack with respect to the wind stream. The relation governing the lateral force requirement with aerodynamic lift generated on the configuration is as follows:

Case 1. Fighter Design

$$(mass * velocity) * \dot{r} = 1/2 * \rho * V^2 * S * C_l$$

Case 2. Sailplane Design

$$\frac{(mass * velocity^2)}{R} = 1/2 * \rho * V^2 * S * C_l$$

Constraint

$$(mass * acceleration\ limit) = 1/2 * \rho * V^2 * S * C_l$$

where

m is the mass of the aircraft,

S is the reference area defining the configuration,

V is the flight speed,

ρ is the density of the air at the altitude of flight, and

C_l is the coefficient of lift for the configuration.

Since m and S can be determined only at the detailed design stage, at the present stage of systems study these two parameters are lumped together to define an additional system functional feature, μ. This parameter that, in a sense is similar to the wing-loading of an aircraft, is defined as

$$\mu = (2 * m) \div (\rho * S)$$

The turn performance requirement and the constraint equation can be recast using this new parameter as follows:

Case 1. Fighter Design

$$\left(\frac{V * C_l}{\mu}\right) \geq \dot{r}_{\max}$$

Constraint

$$V * \left(\frac{V * C_l}{\mu}\right) \leq a_{\max}$$

Case 2. Sailplane Design

$$\left(\frac{C_l}{\mu}\right) \geq \frac{1}{R_{\min}}$$

Constraint

$$V^2 * \left(\frac{C_l}{\mu}\right) \leq a_{\max}$$

Two new system functional features relating the turn performance specification and the corresponding constraint specification have thus been identified. These are:

$\left(V * \frac{C_l}{\mu}\right)$, in the case of fighter design and

$\left(\frac{C_l}{\mu}\right)$, in the case of sailplane design.

As long as the admissible values of these parameters are incorporated in the new design, the performance specification demanded by the user can be ensured in the design. The selection of the admissible range in the values of these two parameters is, however, constrained by (i) The admissible range in the configuration lift coefficient, C_l and (ii) The admissible range in the speed of the configuration, V.

It is seen that the specification on the permissible lateral acceleration constrains the admissible range in the value of the flight speed. The admissible range in the value of the allowable lift coefficient C_l depends upon the aerofoil selected for its lifting surfaces. With these two parameters thus being defined, the compatible range in the value of μ can then be determined. Thus, the technical specification for the detailed design of aircraft or sailplane can be arrived at, in a logical fashion, by following the system study methodology.

For example, in the case of the sailplane design, let the constraint on the tolerance to lateral acceleration be 50 m/s^2. If the maximum expected radius of the thermal was 100 m, then the new design would require a radius of turn less than 100 m.

This requirement would then limit the specification of $\left(\frac{C_l}{\mu}\right) \geq 0.01$.

The limits on lateral acceleration would demand $V^2 * \left(\frac{C_l}{\mu}\right) \leq 50$.

This would limit the maximum velocity of the new design of the sailplane to 70.7 m/s.

In the case of the fighter design, let the constraint specification on the lateral acceleration be 100 m/s^2. If the maximum rate of turn of the adversary was 1 rad/s, the new design would demand

$$\left(\frac{V * C_l}{\mu}\right) \geq 1\ rad/s.$$

The tolerance limit on lateral acceleration would then limit the speed of the fighter to $V \geq 100$ m/s.

The efficacy of the systems study approach is illustrated by the above example. In this procedure, using the laws governing the system in its operating environment, one is able to identify the essential systems functional features needed for its design, without going into the details of the system. One is able to identify subsequently, the admissible range in the values of the constituent parameters of the systems functional features, using the constraint specifications. This admissible range forms the technical specifications for the detailed design.

The main steps needed for conducting the systems study are:

- Knowledge of the environment in which the proposed system should function.
- Knowledge of the physical laws that the system obeys in this environment. Evolving a model for this system. The model should be defined by the minimum set of essential functional features, which are adequate to describe, in an aggregate sense, the functioning of the system in this environment.
- Conduct simulation studies using this model. Vary the system functional features till the response desired by the users is obtained from the system for specified inputs to the system.

Make use of the constraint equations to select the admissible range in the values of the system functional features. The technical specifications for the detailed design of the system are based on this range in the values of the system functional features. Depending upon the constraints within which the designer is seeking solution, various design solutions are feasible for the same set of specifications. All these are valid solutions to the design problem. All feasible solutions should be evaluated for their operational viability.

- The final choice, out of the viable set of solutions, should be made by the user, based on his acceptability criteria.

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CHAPTER 3

GUIDED WEAPONS SYSTEM

The methodology of systems study developed in Chapter 2 is proposed to be applied for the determination of the technical specifications for the design of Tactical Guided Weapons. Since it is known that all design activities start with the enunciation of a need by the user, it is worthwhile digressing a bit and see how the user has arrived at the need for a guided weapon. This background is necessary for understanding the systems study methodology pertinent to guided weapon development.

3.1 SAGA OF WEAPON DEVELOPMENT

From time immemorial, weapons in one form or other have been used by humanity, both for hunting as well as for protection from adversaries. The initial weapons were crude stones and clubs, and the effectiveness of these was found to be limited by the muscle power of the wielder of these implements. The story of weapon development down the ages is, therefore, seen as the saga of one's ingenuity in increasing both the *reach* and the *punch* of the weapons, overcoming the inherent limitation of muscle power. A major landmark in this saga has been the invention of gun powder. This invention, at one stroke, tremendously increased both the reach and the punch of the weapons. Guns of larger bore were developed, shots of heavier mass were employed and, by using greater quantities of gun powder, one could achieve greater effectiveness for these weapons. While these weapons achieved greater successes against static installations and fortification, their effectiveness against moving targets was considerably lower. This was due to the fact that the target was found to have moved away from its original position in the time taken by the shot to reach it. To overcome this shortcoming, the shots were launched by aiming the gun in the anticipated direction of movement of the target. It was hoped that if the prediction of the lead was correct, the target would have moved *just by the requisite amount* to result in an interception with the shot. As can be expected, the success depended largely on the accuracy of the predictions and on the cooperation of the target !

The target, on the other hand, would also be taking every possible effort to avoid the oncoming shot. Thus, whenever the movement of the target was unpredictable, or whenever the target resorted to an evasive manoeuvre, the effectiveness of the gun-launched projectile decreased. A need was therefore foreseen for a weapon that could be *guided* towards the target after it was launched. This requirement implied that, after the weapon was launched against the target, the weapon system should have means for *detecting* any evasive manoeuvre by the target, and based on such detection, the flight path of the weapon should *be corrected* till an interception was achieved with the target. The essential requirements of a guided weapon system are as follows:

- The weapon system should have the means to detect changes in the trajectory of the target.
- Based on such detection, the weapon system should have the ability to assess the correction needed to the trajectory of the weapon to effect an interception with the target.
- The weapon system should have means to implement the assessed course corrections to its trajectory of the weapon.

The corrections to the course of the weapon should be repeated throughout its flight, either continuously or at a periodicity (as in digital control systems) that would be necessary to effect an interception with the target. Highly specialised subsystems are available in modern tactical guided weapon systems to fulfil the above requirements. The characteristics of these subsystems are discussed below.

3.2 SUBSYSTEMS OF GUIDED WEAPON SYSTEM

3.2.1 Observation Device

An observation device is available in all guided weapon systems to track the target and the weapon, and to detect changes in the flight path of the target. The selection of the observation device is based on the characteristics of the signals that emanate from the target which can be used for its detection. Typically, either a radar or an infra-red goniometer is used as an observation device. In very simple systems, however, as in the case of early first generation anti-tank missiles, even optical binoculars have been used !

3.2.2 Guidance System

The course of the weapon is corrected if its current course is not likely to result in an interception with the target. The task of the guidance system is to guide the weapon so as to result in an interception with the target. The magnitude of the required course correction is determined by the chosen *navigation policy*, which makes use of the information extracted by the guidance system from the courses of the target and the weapon. All the navigation policies that are used in modern guided weapon systems fall into two distinct categories.

- Line-of-sight policy
- Proportional navigational policy

3.2.2.1 Line-of-sight policy

In weapon systems following the line-of-sight (LOS) policy, the observation device is fixed to the ground and its beam is locked on to the target by its servo mechanism. A LOS is thus established between the observation device and the target. The beam tracks the target, as the target manoeuvres in three-dimensional space. The LOS policy demands that the weapon always stays on the LOS, so that in course of time, the weapon would intercept the target at the farther end of the LOS. To achieve this objective, navigation commands are sent to the weapon to generate lateral acceleration on it, which is proportional to the offset distance of the weapon from the LOS and directed towards the LOS.

3.2.2.2 Proportional navigational policy

In weapons adopting a proportional navigation policy, however, the observation device is located on the weapon itself and moves with the velocity of the weapon, as the weapon manoeuvres in three-dimensional space. The beam of the observation device, which is slaved to the target, now rotates in space under the influence of the relative velocity of the target with respect to the weapon. The LOS thus has an angular rate. The proportional navigation policy demands that the angular rate of change of the flight path of the weapon is proportional to the angular rate of change of the LOS.

The functions of detection and the subsequent navigation of the weapon are so much interdependent that these are clubbed together in the *guidance system* of the weapon. The *beam rider* and the *command to line-of-sight (CLOS/ACLOS)* weapons are typical examples of systems following a LOS policy, while all *homing* missiles follow either the proportional navigation policy or its variants.

Correction to the course of the weapon is implemented by generating forces *lateral to its heading.* The magnitude of the lateral force needed to effect such course correction is decided by the chosen guidance policy. The lateral force is generated on the weapon, either by vectoring the propulsive thrust in the required lateral direction (Thrust-Vector Controlled Weapons), or by generating the required lateral force on the weapon by aerodynamic means. In the latter, the weapon configuration is set at the required angle of attack with respect to the wind-stream to generate the required lateral lift force.

Guided weapons are usually symmetric configurations and, therefore, have a natural tendency to align with the direction of the prevailing wind and fly at zero angle of attack. It is therefore necessary to apply a moment about its centre of gravity to set the configuration at the required angle of attack, that is needed for generating the required lateral lift force.

3.2.3 Control System

A control system is used to generate the required lateral force either by vectoring the nozzle of the thrusting device, as in thrust vector-controlled weapons, or by setting the configuration at the required angle of attack. In aerodynamic configurations, the lateral force is generated by deflecting the aerodynamic control surfaces in the appropriate direction. This produces the necessary control force that would, in turn, generate the required control moment about the centre of gravity of the weapon.

In aerodynamically stable configurations, the neutral point of the configuration lies aft of its centre of gravity. Therefore, when the control moment is applied, the additional aerodynamic lift force generated on the configuration as a consequence would produce a counteracting aerodynamic moment about the centre of gravity. Under the influence of these two opposing moments, the weapon would execute *short period* oscillations (also known as *weather cock* oscillations) before settling down to the demanded angle of attack. There is a unique functional relationship between the angle through which the control surface needs to be deflected, and the lateral lift force that is required to be generated on the configuration. The act of deflecting the control surfaces through this unique angle is achieved by control actuators. These are either hydraulic control actuators or pneumatically actuated ones. In lightly loaded systems, however, even electrically operated control systems have been used.

It is seen that, under the influence of the two opposing moments, the weapon would execute short period oscillations. Hence the lateral acceleration demanded by the guidance policy would take a *finite time* to be generated on the weapon. This results in the weapon *lagging* behind the control demand. As a consequence of this control lag, the weapon would not follow the demanded path and therefore miss the target by a finite distance. The least amount by which the weapon misses the target is known as *miss distance.*

3.2.4 Warhead

To increase the effectiveness of the weapon overcoming the inherent miss distance, the warhead in the weapon is selected so as to have an adequate *zone of effectiveness* around it. This zone of effectiveness of the warhead is defined by its *lethal radius* which is a *statistical distance* around the warhead, in which an *assured level of lethality* is achieved. The assured level of lethality is a statistical figure defined by the *Single Shot Kill Probability (SSKP)* of the warhead. The lethal radius of the warhead is dependent on the type of warhead, the type of explosive and the mass of its explosive content. The higher the explosive content the higher is its zone of effectiveness. The desired *SSKP* of the weapon is said to have been achieved when the final miss distance achieved by the weapon is within the lethal radius of the warhead. A weapon system with a higher miss distance, would require a warhead of a heavier mass to achieve the desired level of lethality. Consequently, this leads to a heavier weapon. To make the weapon lighter and still have an assured level of lethality, it is necessary to *minimise* the miss distance by proper selection of the functional features defining the weapon system.

The functional parameters defining these two subsystems of the weapon are selected based on systems study, to achieve a minimum weight solution for the weapon system. The procedure for conducting such a systems study is developed in this chapter.

3.3 SYSTEMS STUDY

3.3.1 System Model

To conduct systems study, it is necessary to have a system model that reflects the designer's holistic perception of the weapon system. The systems designer perceives the target and the weapon as two point masses moving in three-dimensional space. While the motion of the weapon is under the command of the chosen guidance policy, the motion of the target is not under the control of the systems designer. In fact, the motion of the target is under the control of the adversary and is therefore an independent input for conducting the systems study. A typical schematic diagram for the system model, reflecting this perception of the systems designer is shown in Fig. 3-1. The minimum set of essential system elements which are needed for defining the performance of the system, in an aggregate sense, is depicted in the diagram. Thus,

(a) The observation device is depicted by a comparator that compares the information about the target and the weapon. The output from the comparator is the information on which the guidance policy of the weapon system is based.

(b) The guidance block takes in the information from the comparator and outputs the lateral acceleration that is required to be generated on the weapon.

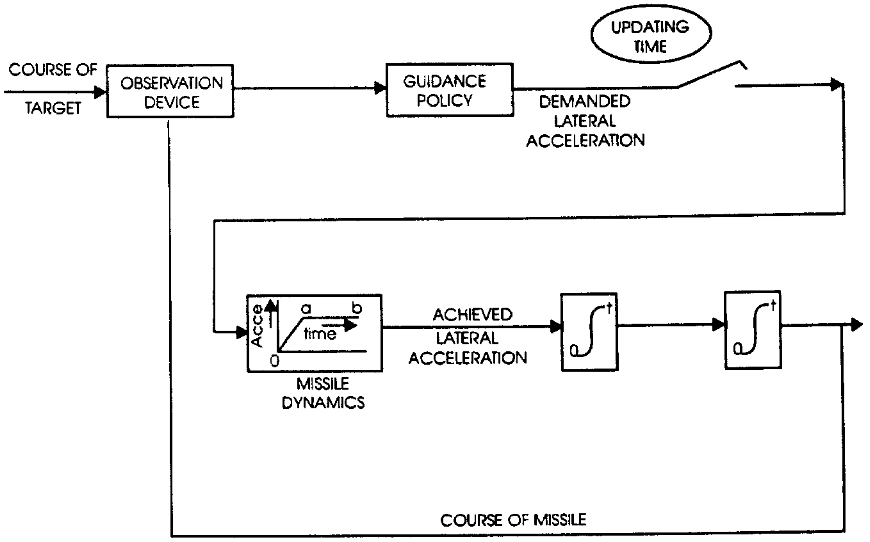

Figure 3-1. Schematic of system model

(c) The need for command updating is represented by a command updating switch that completed the circuit at a periodicity of t_{update}.

(d) The control subsystem is *assumed* to be the fastest in the entire system. Therefore every guidance demand received by it (whenever the command updating switch is closed) is assumed to have been *executed instantaneously* by the control subsystem. This would imply a step deflection of the control surfaces (or the thrusting nozzle), which results in a step generation of the control force on the weapon. The response of the weapon to such a *step control demand* would be the *short period response* of the system.

(e) The next block represents the dynamics of the weapon configuration in its short period mode. The short period characteristic is represented by its rise time, t_{rise}. The weapon would therefore generate the demanded lateral acceleration with a finite time lag, t_{rise}.

(f) The next two blocks are simple integrators representing the inertia of the configuration in its short period mode. After the first integration we get the velocity of the weapon and after the next integration we get the position of the weapon in three-dimensional space. This position information is fed back to the comparator, thus completing the loop.

This model of the guided weapon system is known as the *Outer Loop* model. Systems study is conducted using this model of the outer loop. The objectives of the systems study is to determine the *kernel of essential functional characteristics* of the outer loop, which leads to the achievement of acceptable final miss distance of the weapon, for all possible attack trajectories of the target.

3.4 INERTIAL FRAME OF REFERENCE

It is obvious from the system model that there is a need to define the courses of the target and the weapon for conducting systems studies. Since the target and the weapon are two points moving in three-dimensional space, their trajectories can be represented conveniently by using an inertial frame of reference. A typical inertial frame of reference is illustrated in Fig. 3-2.

In this reference system, the position of every point in three-dimensional space is represented by its position vector, which denotes the distance of the point from the fixed origin of the chosen reference system. In the figure, a right-handed reference system OXYZ is selected with its origin at O. The orientation of the position vector OP uniquely fixes the position of point P in three-dimensional space. In general, three angles are needed to fix the orientation of the position vector in this system of reference. These are the three angles subtended at the origin by the position vector, with the three reference axes. The cosines of these angles are used for further computation. These *direction cosines* satisfy the identity that the sum of their squares is unity. Thus, if l, m, n, are the three direction cosines of the position vector OP, then

$$l^2 + m^2 + n^2 \equiv 1 \tag{3.1}$$

In practice, the position vector can be defined uniquely by two angles only, namely, the angle of azimuth and the angle of elevation. To define these two angles, it is necessary to find a vertical plane containing the position vector. It is easy to determine this plane as the vertical plane through the origin containing the position vector. In Fig. 3-2, OP is the position vector that lies in the vertical plane OPN passing through the origin. The angle Φ that the vertical plane makes with the axis OX (in a positive sense of rotation of OX) is termed the azimuth angle. The angle Θ that the position vector OP makes with the OZ axis (in a positive sense of rotation of OZ) is the elevation angle. The direction cosines of the position vector can be obtained from these two angles as follows:

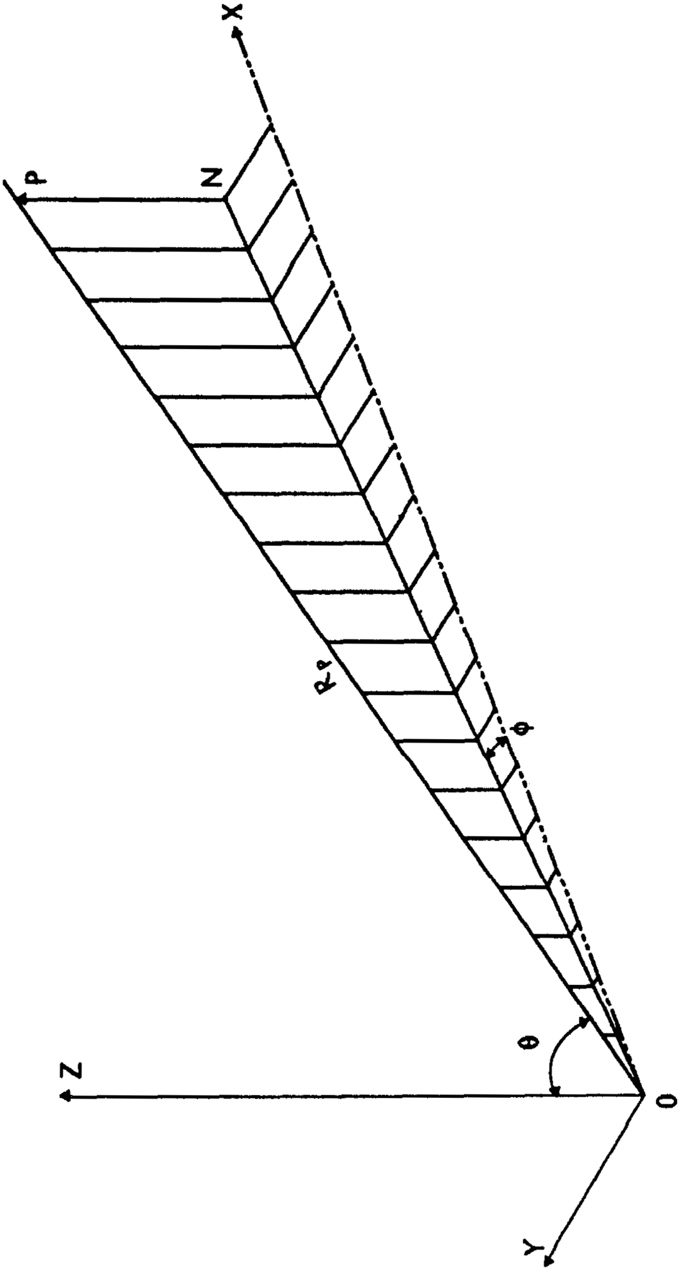

Figure 3-2. Inertial frame of reference

$$\left.\begin{aligned} l &= Sin\Theta * Cos\Phi \\ m &= Sin\Theta * Sin\Phi \\ n &= Cos\Theta \end{aligned}\right\} \quad (3.2)$$

It is seen that by the use of the inertial set of reference, the positions of the target and the weapon can be uniquely specified. Thus, the courses of the target and the weapon, the inputs needed for conducting the systems study, can be defined in this inertial frame of reference.

The objective of systems study is to choose the functional features defining each of the elements in the system model in such a way as to make the final miss distance a minimum, *for all possible attack trajectories of the target.* However, the trajectories of the target are not definable, since the motion of the target is under the command of the adversary. The course of the target is therefore the independent input for conducting the systems study. To make the systems study fruitful, it is necessary that the target attack trajectories used in the systems study be represented as close to reality as possible. This is a *guesing game.* In this type of game, a good maxim would be to *credit the adversary with as much intelligence and capability as we are ourselves capable of.*

Usually, a good guess is based on *intelligence* information that have been gathered about the performance capabilities of the target. Specifically, information about the speed of the target, its manoeuvre capabilities and the terrain over which the attack is likely to be launched is the prequisite needed for modelling the likely attack trajectories of the target. This information, together with certain basic principles of flight, are made use of in generating realistic target attack trajectories. A method for deriving these trajectories is developed[1] in the next paragraph.

3.5 TARGET TRAJECTORY MODEL

It is well known from the theory of flight that:

(a) Target can execute a manoeuvre only by generating manoeuvre forces in directions lateral to its instantaneous flight velocity vector.

(b) Aerodynamic lift force for executing a manoeuvre can be generated by the flying configuration only in directions perpendicular to its instantaneous flight velocity vector. Therefore, aerodynamic manoeuvre lift force can be generated only in a *Pencil of Planes* containing its *instantaneous velocity*

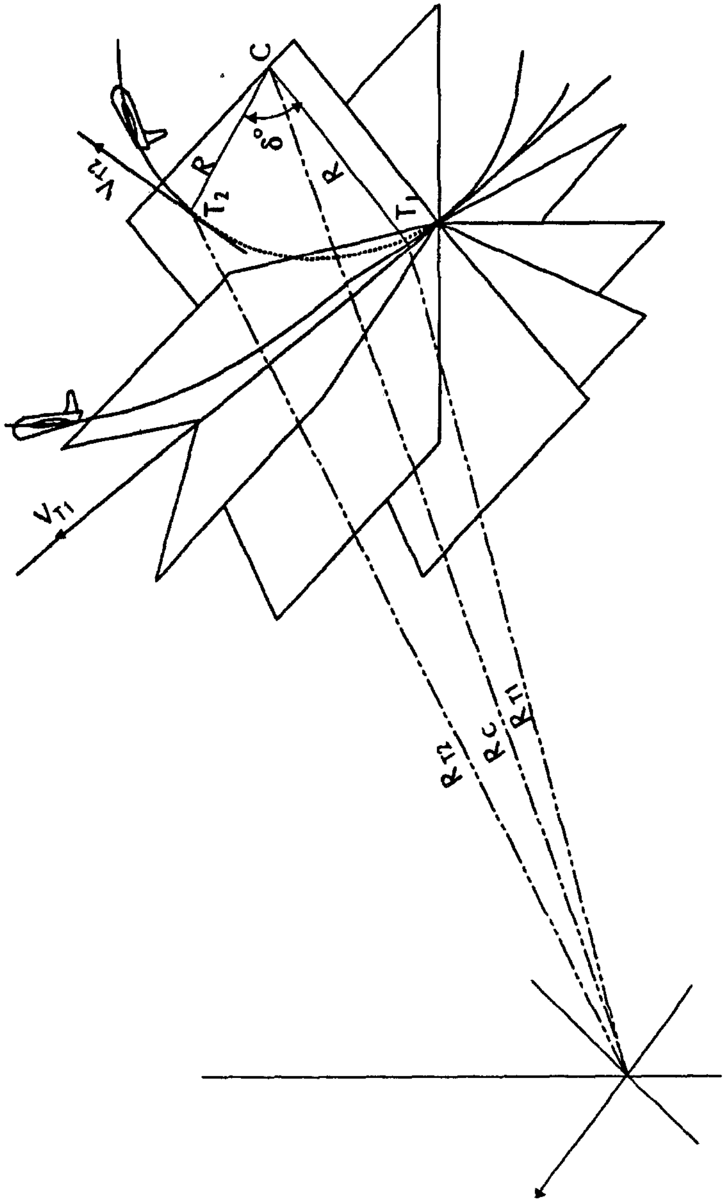

Figure 3-3. Pencil of target manoeuvre planes

vector. Fig. 3-3 depicts such a pencil of planes in which the target can choose to manoeuvre.

(c) A manoeuvre can be executed by the target in any plane belonging to this pencil of planes. The manoeuvre executed by the target in this chosen plane is constrained both by the maximum lift generating capability of the configuration and also by the physical tolerance of the contents of the configuration to withstand sustained lateral acceleration.

These three facts are adequate to develop a model to determine all the possible target attack trajectories.

3.5.1 Target Manoeuvre Planes

Let l_{vt}, m_{vt}, n_{vt} be the direction cosines of the instantaneous velocity vector of the target and let l_{nt}, m_{nt}, n_{nt} be the direction cosines of the vector normal to the manoeuvre plane (chosen out of this pencil of planes) that contains the *instantaneous velocity vector* of the target. The requirement that these two vectors should be perpendicular to each other leads to the condition:

$$l_{vt} * l_{nt} + m_{vt} * m_{nt} + n_{vt} * n_{nt} = 0 \tag{3.3}$$

The direction cosines of the normal vector to the chosen manoeuvre plane also satisfy the identity

$$l_{nt}^2 + m_{nt}^2 + n_{nt}^2 \equiv 1 \tag{3.4}$$

It can thus be noticed that out of the three direction cosines defining the normal vector to the chosen manoeuvring plane, the above two equations allow only one of them to be chosen arbitrarily. The arbitrary selection of any one of these three direction cosines by the adversary makes the trajectory of the target unpredictable. This unpredictable behaviour can be modelled by using *standard probabilistic methods* to select any one of these three direction cosines.

The adversary has three options. He can arbitrarily select l_{nt}, m_{nt}, *or* n_{nt}. Having chosen one out of these three direction cosines, the other two direction cosines are uniquely fixed by the above two conditions. Thus,

Case 1

Let the pilot select l_{nt}. He can make this choice only if l_{vt} is not equal to ± 1. This implies that the adversary can make a choice of l_{nt}

only if the target was not flying, at that moment, in planes parallel to the Y-Z plane.

Let $l_{nt} = p; \qquad -1 \le p \le +1$

Then, either

$$\left.\begin{aligned} m_{nt} &= \frac{-p * l_{vt} * m_{vt} \pm n_{vt} * \sqrt{1 - p^2 - l_{vt}^2}}{1 - l_{vt}^2} \\ n_{nt} &= \pm\sqrt{1 - p^2 - m_{nt}^2} \end{aligned}\right\} \tag{3.5a}$$

or

$$\left.\begin{aligned} n_{nt} &= \frac{-p * l_{vt} * n_{vt} \pm m_{vt} * \sqrt{1 - p^2 - l_{vt}^2}}{1 - l_{vt}^2} \\ m_{nt} &= \pm\sqrt{1 - p^2 - n_{nt}^2} \end{aligned}\right\} \tag{3.5b}$$

Case 2

Let the pilot select m_{nt}. He can make this choice only if m_{vt} is not equal to ± 1. This implies that the adversary can make a choice of m_{nt} only if the target was not flying at that moment, in planes parallel to the X-Z plane.

Let $m_{nt} = p; \qquad -1 \le p \le +1$

Then, either

$$\left.\begin{aligned} l_{nt} &= \frac{-p * m_{vt} * l_{vt} \pm n_{vt} * \sqrt{1 - p^2 - m_{vt}^2}}{1 - m_{vt}^2} \\ n_{nt} &= \pm\sqrt{1 - p^2 - l_{nt}^2} \end{aligned}\right\} \tag{3.6a}$$

or

$$\left.\begin{aligned} n_{nt} &= \frac{-p * m_{vt} * n_{vt} \pm l_{vt} * \sqrt{1 - p^2 - m_{vt}^2}}{1 - m_{vt}^2} \\ l_{nt} &= \pm\sqrt{1 - p^2 - n_{nt}^2} \end{aligned}\right\} \tag{3.6b}$$

Case 3

Let the pilot select n_{nt}. He can make this choice only if n_{vt} is not equal to ± 1. This implies that the adversary can make a choice of n_{nt} only if the target was not flying, at that moment, in planes parallel to the X-Y plane.

Let $n_{nt} = p; \qquad -1 \leq p \leq +1$

Then, **either**

$$\left.\begin{aligned} l_{nt} &= \frac{-p * n_{vt} * l_{vt} \pm m_{vt} * \sqrt{1 - p^2 - n_{vt}^2}}{1 - n_{vt}^2} \\ m_{nt} &= \pm\sqrt{1 - p^2 - l_{nt}^2} \end{aligned}\right\} \tag{3.7a}$$

or

$$\left.\begin{aligned} m_{nt} &= \frac{-p * n_{vt} * m_{vt} \pm l_{vt} * \sqrt{1 - p^2 - n_{vt}^2}}{1 - n_{vt}^2} \\ l_{nt} &= \pm\sqrt{1 - p^2 - m_{nt}^2} \end{aligned}\right\} \tag{3.7b}$$

The above equations enable one to uniquely determine the direction cosines of the normal vector to the selected target manoeuvring plane. It is noticed that the arbitrary selection of one of these three direction cosines of the normal vector leads to the determination of four orthogonal planes in which the target can manoeuvre, depending upon the subsequent selection of either the '+' or the '–' sign in the above equations.

Once the selection of the manoeuvre plane has been made, the target will manoeuvre only in this plane till the next instant of selection of another plane for manoeuvring. In this chosen manoeuvring plane, the target will describe a circular arc under the action of the lateral manoeuvring force. The radius of the circular arc is determined by the lateral acceleration demanded by the adversary for manoeuvring the target. However, the maximum possible manoeuvre is constrained both by the tolerance limit of the contents of the target to lateral acceleration and the manoeuvre capability of the configuration. Let the lateral acceleration selected for manoeuvring be (n*g). The radius of the manoeuvring arc is then given by

$$R = \frac{V_T^2}{n * g} \tag{3.8}$$

The direction of the radius vector is uniquely determined by the two requirements, that the radius vector should lie in the manoeuvre plane and should also be perpendicular to the instantaneous velocity vector. These two requirements are ensured by the condition that the radius vector, the normal vector to the manoeuvring plane and the instantaneous velocity vector should form a right-handed triad (Fig. 3-3). The unit vector, in the direction of the radius, is given by

$$\hat{R} = \frac{\hat{V}_T \times \hat{N}}{|\hat{V}_T \times \hat{N}|} \tag{3.9}$$

Let C be the centre of the manoeuvre circle. The position vector of the centre of the manoeuvre circle is given by

$$\vec{R}_C = \vec{R}_{T_1} + \hat{R} * \text{R} \tag{3.10}$$

Till a selection of another manoeuvring plane is made by the adversary, the target would be describing a circular arc in this selected manoeuvring plane with its centre at *C* and radius *R*. Let the radius vector sweep an arc during this interval of time, subtending an angle δ at the centre. The new position vector of the target, at the end of this interval of time is got by the following two-step process:

- Moving through a distance of $|\vec{R}| * (1\text{-Cos}\ \delta)$ in a direction parallel to the radius vector, and by
- Moving through a distance $|\vec{R}| * \text{Sin}\ \delta$ in a direction parallel to the instantaneous velocity vector.

Thus, the new position vector of the target at the end of this time is given by

$$\bar{R}_{T_2} = \bar{R}_{T_1} + \hat{R}\, R^* \,(1\text{-Cos}\delta) + \hat{V}_T * R * \text{Sin}\delta \qquad (3.11)$$

The unit vector in the direction of the velocity of the target at its new position is determined by the requirement that this unit velocity vector, together with the radius vector at the new position of the target and the unit normal vector to the manoeuvring plane should form a right-handed triad. Thus, the unit velocity vector at the new position of the target is given by:

$$\hat{V}_T = \hat{N} X \frac{\left(\bar{R}_{T2} - \bar{R}_C\right)}{R} \qquad (3.12)$$

The velocity vector at the new position of the target is obtained as:

$$\bar{V}_T = V_T * \hat{V}_T \qquad (3.13)$$

The above equations enable one to evolve realistic target trajectories for use in the system simulation studies. The selection of the instant for initiating a manoeuvre by the target and the selection of the magnitude of the lateral acceleration needed for its manoeuvre can both be implemented by standard probabilistic methods. In this way, the unpredictable character of the motion of the target can be represented realistically for use in systems studies.

3.6 DYNAMIC CHARACTERISTICS OF THE WEAPON

The short period dynamics of the configuration of the weapon is represented in the systems study model by its rise time, t_{rise}. Every time a control demand is received by the system, the lateral acceleration is generated in the rise time of the configuration, t_{rise}. During this period, the magnitude of the lateral acceleration demanded on the configuration would be generated either in an oscillatory manner, if the weapon is under-damped, or in an exponential manner, if it is over-damped. The exact nature of the curve can be determined only during the detailed design stage of the aerodynamic configuration. At the present stage of systems study however, it is sufficient if this lag is represented as a ramp function that attains its demanded value in the rise time and then stays saturated at the demanded value till the next command update is received.

3.6.1 Command Updating Time

It was explained earlier that since the configuration takes a finite time, t_{rise}. to generate the demanded lateral acceleration, the weapon would not follow the path demanded by the guidance policy for intercepting the target. There is, therefore, a need to update the guidance commands at a periodicity, t_{update}. What is the basis for determining its periodicity? The periodicity is determined by the following considerations:

Upper Bound of t_{update}

Since the control subsystem is assumed to be the fastest in the whole system, the command to the control system is assumed to have been executed instantaneously. Thereafter, it takes a finite time, t_{rise} for the system to respond to the demand and to generate the demanded acceleration. Because of this lag, the resultant trajectory followed by the weapon would not be the trajectory demanded by the guidance system. *The deviation* of the resultant trajectory from the demanded trajectory *should be assessed*, before the next command update can be generated by the guidance subsystem. The time required for all the activities needed for this assessment should form the upper bound of t_{update}.

Lower Bound of t_{update}

The lower bound however should be t_{rise} itself, which is the time required for the system to respond to the previous command.

3.7 SYSTEMS SIMULATION

All the essential system functional features needed for describing the performance of the weapon system (as perceived by the systems designer who takes a holistic view of the encounter between the target and the weapon) have been represented in the systems model developed in this chapter. A method has also been developed for representing the attack trajectories of the target in as realistic a manner as possible. This system model can now be used for the selection of the compatible ranges in the values of the kernel of essential system functional features that are needed for the design of the guided weapon system.

There is, however, a slight difference in the usage of the system model developed here, from the way it was used in the example cited in Chapter 2. In the example regarding the *turn performance* of aircraft cited, the turn performance expected from the aircraft was stated in precise quantitative terms. The laws governing the flight of the aircraft could be used for arriving at the precise set of the essential system functional features. In the case of guided weapon design however the precise miss distance expected out of the system is not definable *apriori.* The endeavour of the systems designer should therefore be to select the systems functional features in such a manner as to make the final miss distance of the weapon as close to the target as possible, for all possible attack trajectories of the target. This would require that the influence of the parameters defining the guidance subsystem and the short period dynamics of the configuration on the miss distance be assessed during systems study. The set of parameters defining the essential functional features of these two interacting subsystems is selected based on an analysis of the influence of these parameters on the miss distance. The set of system functional features that needs to be selected from such an analysis is the following:

- The parameters defining the guidance system.
- The ratio of the speed of the weapon to that of the target.
- The rise time of the weapon in its short period mode.
- The frequency at which the commands need to be updated.
- The longitudinal acceleration needed for the propulsion of the weapon.
- The lateral acceleration demanded on the weapon by the chosen guidance policy.

Before undertaking the simulation studies using this systems model, the guidance policy should be selected. The considerations for the choice of the command guidance policy is discussed in Chapter 4, and that of the proportional navigational policy in Chapter 5.

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1. Balakrishnan, R. Aerodynamic configuration design for cruciform missiles. IIT, Madras, 1983. Ph.D. Thesis.

CHAPTER 4

SYSTEMS STUDY FOR COMMAND GUIDED WEAPON SYSTEMS

Command guided weapon systems are some of the earliest forms of guided weapon systems developed during World War II. These systems used a very simple form of guidance policy which followed the natural tendency of human beings to correct errors whenever they occured. In first generation wire guided anti-tank missiles, where this guidance policy was used, the soldier trained the cross-wires of a binocular on the enemy tank and thus established a line-of-sight (LOS) between his eye and the tank. He then launched the missile towards the tank such that, in the initial part of its trajectory, the missile was also seen in the field of view of the binocular. Then, noticing the position of the missile with reference to the centre of the cross-wires, the soldier sent commands to the missile through its trailing wire to manoeuvre the missile and bring it to the LOS. It was expected that if the missile could be held on the LOS long enough, then it would intercept the tank which was at the other end of the LOS. Thus, here was a weapon system that followed the natural human reactions and therefore found ready acceptance by the user. But it was soon realised that holding the missile on the LOS was a taxing exercise for the soldier and consequently, the rate of hits achieved by such a system was not high enough. Various modifications to the basic policy were therefore necessitated to increase the rate of kill. In its modified form, this policy finds applications in a number of current short-range beam rider weapon systems and in command to line-of-sight (CLOS) weapon systems. In this chapter, it is proposed to develop a system study methodology for arriving at the *kernel of system functional features* that are essentially needed for the design of a weapon system following the command guidance policy. It is proposed to develop first the basic beam rider policy and subsequently modify it to arrive at the CLOS policy. However, before this is attempted, understanding the view point of the systems designer is essential.

4.1. COMMAND GUIDANCE POLICY—SYSTEM DESIGNER'S POINT OF VIEW

The systems designer perceives the basic command guided weapon system essentially as an interaction between three points.

The first, representing the target, is seen to be following a course which is not under the control of the systems designer. The second, representing the weapon, moves in three-dimensional space following the guidance commands issued to it from the third point situated on the ground, where an observation device is located. By means of servomechanisms, the observation beam is slaved to the target and thus an LOS is established. This beam rotates in three-dimensional space while it follows the target. The displacement of the weapon from the beam is noted and, from time to time corrections *proportional to the offset distance* of the weapon from the LOS are sent to the weapon.

This perception of the systems designer forms the basis of the systems model which is used for systems studies.

4.2 SYSTEM MODEL

A typical scenario of an encounter between the weapon and the target is illustrated in Fig. 4-1. In the figure, O is the location on the ground of the observation device. Usually, either a radar or an infra-red goniometer is used as an observation device. The points T and M represents the target and the weapon respectively. $\vec{V}_T$ and $\vec{V}_M$ are the velocity vectors of the target and the weapon. The straight line OT represents the LOS and MP represents the offset distance of the instantaneous position of the weapon from the LOS. The basic command guidance policy demands lateral accelerations on the weapon which is proportional to the magnitude of the offset distance of the weapon from the LOS and directed towards the LOS. The *magnitude* of the demanded lateral acceleration is given by

$$A = -G * MP \tag{4.1}$$

where, G is the guidance gain measured in units of accelerations per meter offset.

It is seen that under such a policy the weapon would execute simple harmonic motion about the LOS. This oscillatory mode, known as the *Weave Mode*, has a frequency given by

$$\omega_{weave} = \sqrt{G} \tag{4.2}$$

To damp out the oscillatory behaviour, lateral acceleration proportional to the rate of change of the offset distance is also demanded additionally. The *magnitude* of the lateral acceleration demanded on the weapon by the modified guidance policy is given by

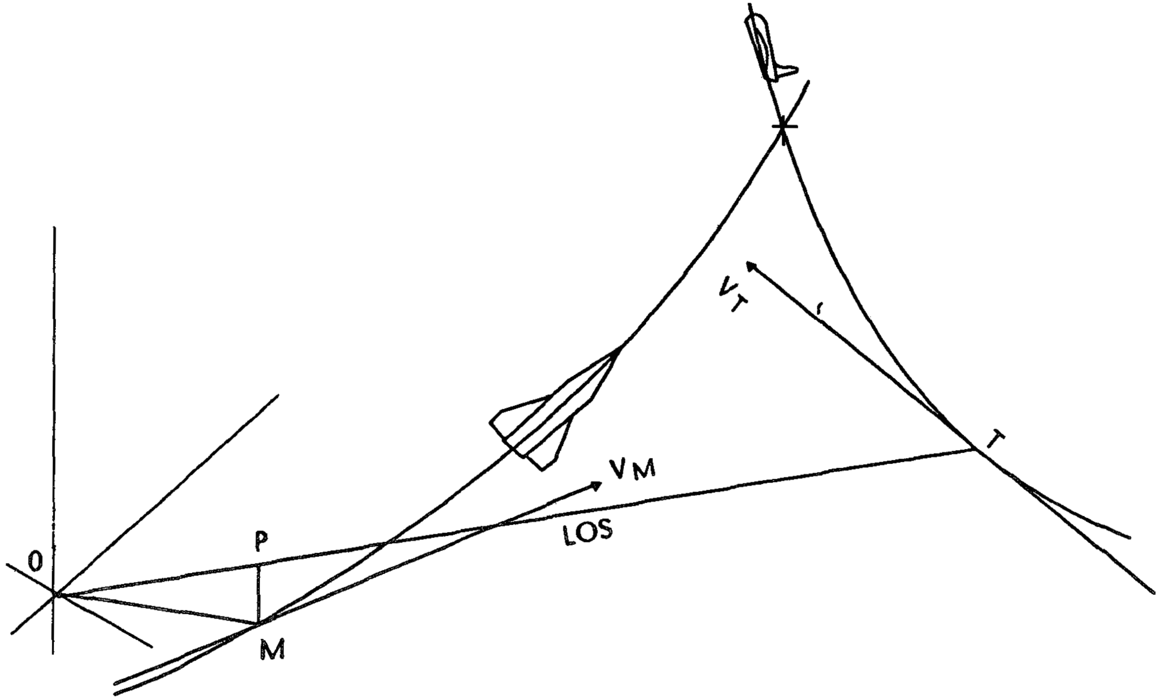

Figure 4-1. Line-of-sight guidance scheme

$$A = -G * \left(MP + T * \frac{d(MP)}{dt} \right) \tag{4.3}$$

where, T is a time parameter chosen to provide a specified value of damping ratio in the weave mode. The damping ratio is related to T by

$$\zeta_{weave} = \sqrt{G} * \frac{T}{2} \tag{4.4}$$

4.2.1 Effect of Weave Mode Damping

The effect of incorporating damping in the weave mode is illustrated in Fig. 4-2. As expected, without the damping term, the

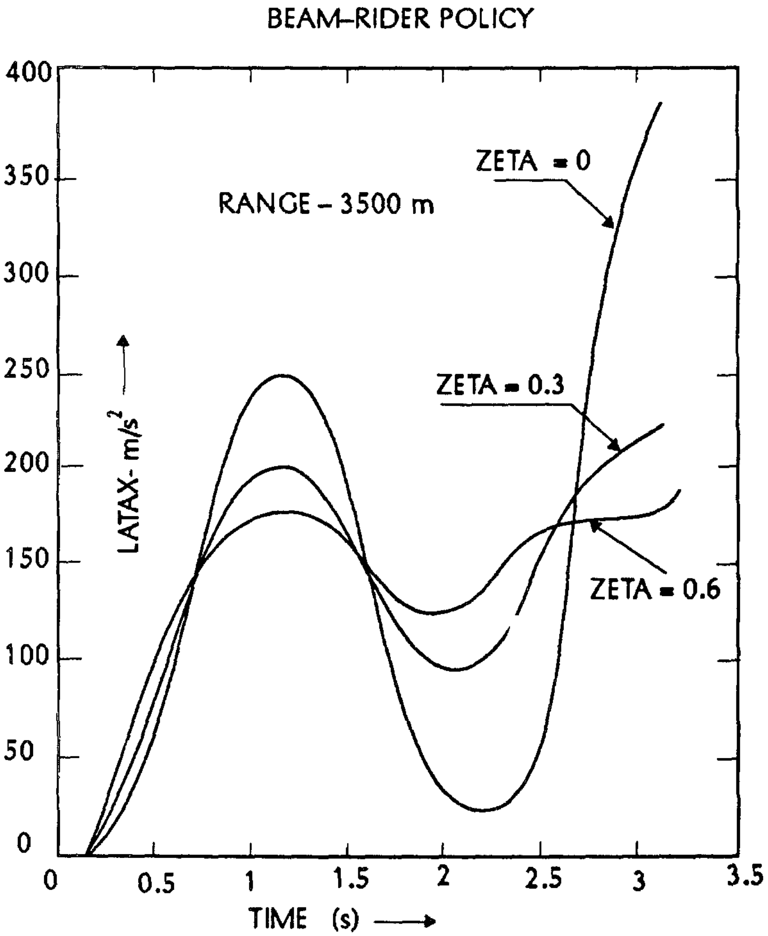

Figure 4-2. Effect of weave mode damping on lateral acceleration

lateral acceleration demanded on the weapon is oscillatory. This is shown by Zeta=0.0 curve in figure. When the damping ratio is increased to Zeta=0.3 and 0.6, the demands on the lateral acceleration are observed to be significantly lower. It is therefore a good practice to increase the damping ratio in the weave mode.

4.2.2 Reference Coordinate System

The guidance of the weapon demands that the trajectories of the target and the weapon be observed and the corrections to the course of the weapon be based on an assessment of how close the weapon was following its demanded trajectory. Since, two moving points are being dealt with, the trajectories of the target and the weapon are best observed in an inertial frame of reference. In command guided systems, since there is a ground fixed point in the location of the observation device, this point is chosen conveniently as the origin of a three-dimensional reference coordinate system. The *orientation* of a right handed triad of three-dimensional unit vectors, located at this origin, is made under the following consideration:

At the beginning of the encounter, the observation device scans the three-dimensional space around it and searches for the target. When the target is sighted, the beam of the observation device is locked on to the target by its servo system, thus establishing LOS between the observation device and the target. The observation device then tracks the target and when the target is within the range, the weapon is launched against it. At the instant of weapon launch, taken as the start of time reference, the reference triad of unit vectors is rotated about the origin till the instantaneous LOS lies in the XOZ plane of the triad. *Having thus aligned the vertical plane of the triad to contain the LOS at the start of time, the orientation of the right handed three-dimensional coordinate reference frame is fixed.* At subsequent intervals of time, the position vectors of the target and the weapon are determined only in this frame of reference. A typical three-dimensional right handed reference axes system, at the instant of the launch of the weapon, is shown in Fig. 4-3.

The initial position vector of the target is OT_0, which is at an azimuth angle of $\Phi_{R_{T_0}} = 0°$, and at a finite elevation angle of $\Theta_{R_{T_0}}$. The Cartesian coordinates of the position of the target, in the chosen three-dimensional coordinate reference system, is given by:

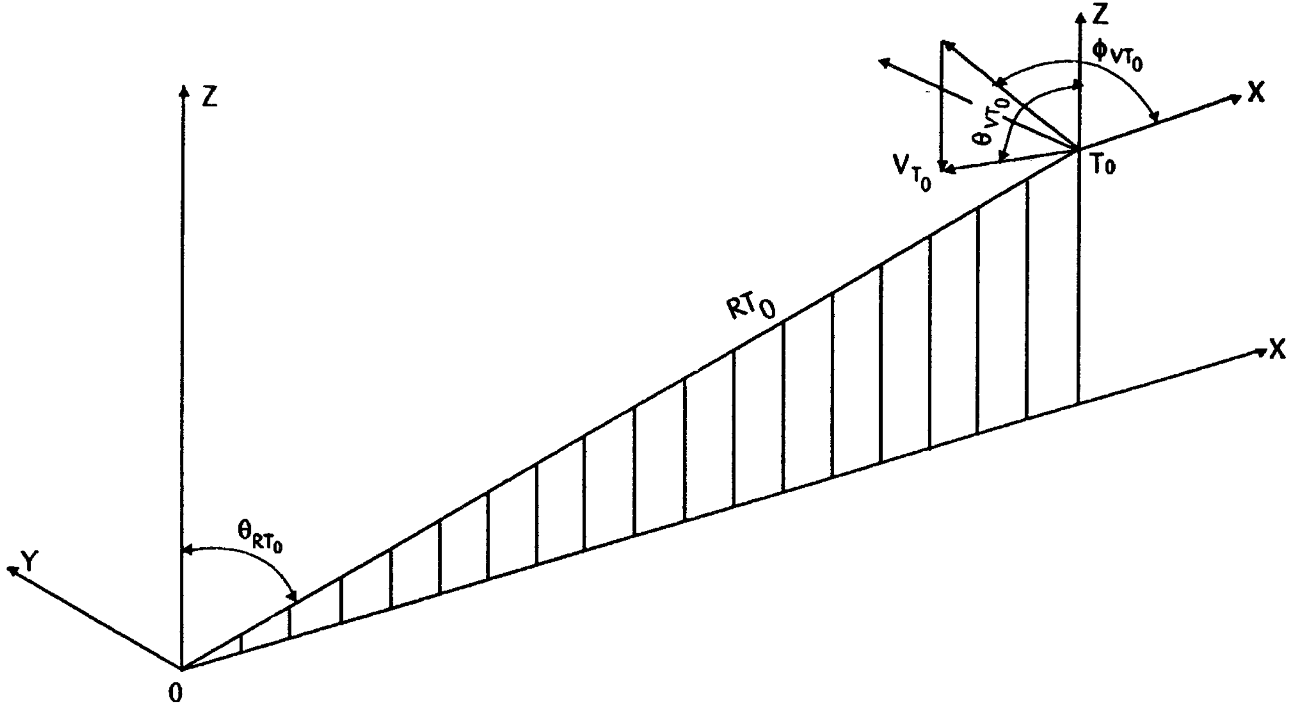

Figure 4-3. Inertial reference at the instant of weapon launching

$$\left.\begin{aligned} X_{T_0} &= R_{T_0} * \mathrm{Sin}\Theta_{R_{T_0}} \\ Y_{T_0} &= 0.0 \\ Z_{T_0} &= R_{T_0} * \mathrm{Cos}\Theta_{R_{T_0}} \end{aligned}\right\} \qquad (4.5)$$

where, R_{T_0} is the magnitude of the initial position vector of the target.

In general, the velocity vector of the target, $\vec{V}_{T_0}$, will be at an azimuth angle $\Phi_{V_{T_0}}$ and at an elevation angle $\Theta_{V_{T_0}}$. These angles are easily defined by constructing a reference triad at T_0 which is parallel to the reference coordinate axes at the origin. Thus, the velocity vector will have its Cartesian components as

$$\left.\begin{aligned} X_{V_{T_0}} &= V_{T_0} * \mathrm{Sin}\Theta_{V_{T_0}} * \mathrm{Cos}\Phi_{V_{T_0}} \\ Y_{V_{T_0}} &= V_{T_0} * \mathrm{Sin}\Theta_{V_{T_0}} * \mathrm{Sin}\Phi_{V_{T_0}} \\ Z_{V_{T_0}} &= V_{T_0} * \mathrm{Cos}\Theta_{V_{T_0}} \end{aligned}\right\} \qquad (4.6)$$

If, $\Phi_{V_{T_0}}$ is 180°, the target is said to be an approaching target and when $\Phi_{V_{T_0}}$ is 0°, then the target is said to be a receding target. In both these cases, however , the trajectory of the target would lie in the initially defined vertical plane, XOZ. At any other azimuth angle, however, the target is said to be a crossing target and its path would not lie in the initially defined plane XOZ.

4.3 MATHEMATICAL MODEL

At any interval of time after the launch of the weapon, the scenario would be as shown in the Fig. 4-4. The instantaneous position vector of the target is OT. This is given by

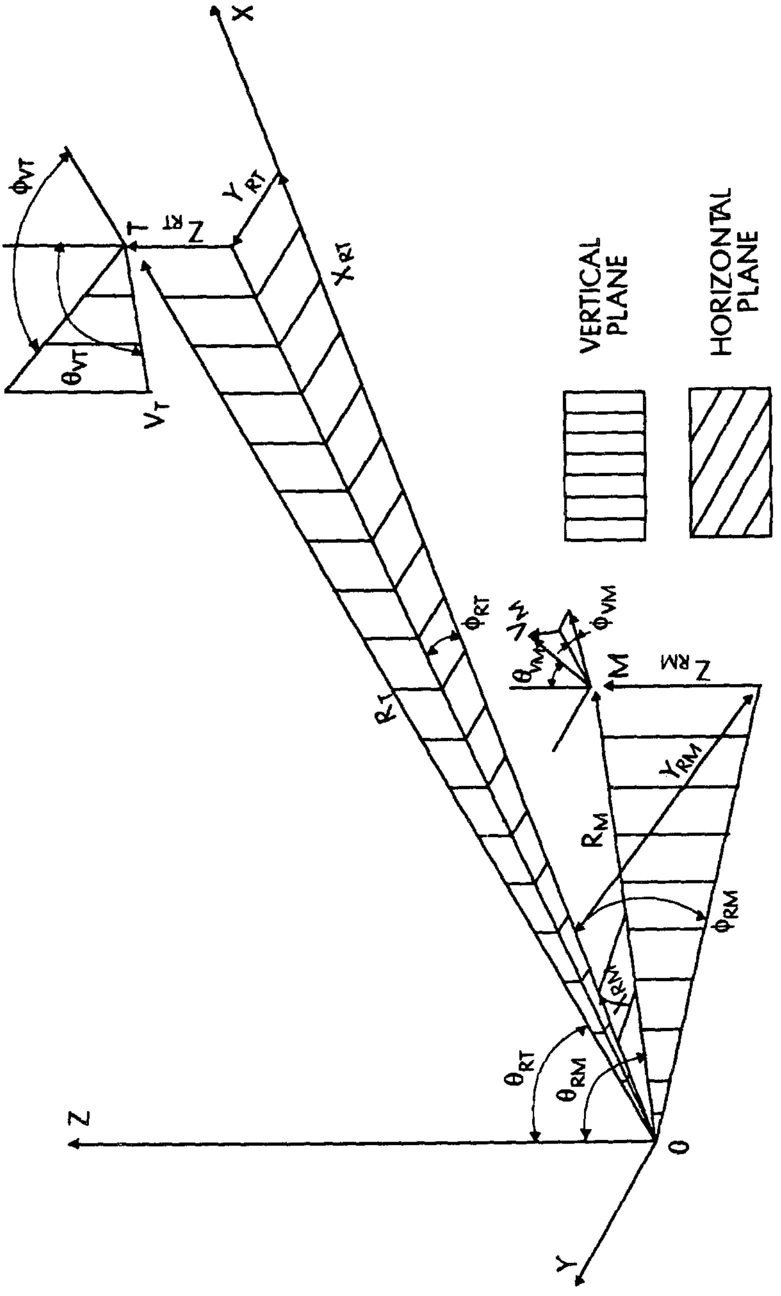

Figure 4-4. Position vectors at any instant of time

$$\vec{R}_T = R_T * \left(\hat{i} * l_{R_T} + \hat{j} * m_{R_T} + \hat{k} * n_{R_T}\right) \qquad (4.7)$$

where, the direction cosines of the vector $\vec{R}_T$ are $l_{R_T}, m_{R_T}, n_{R_T}$

The instantaneous position vector of the weapon is OM which is given by

$$\vec{R}_M = R_M * \left(\hat{i} * l_{R_M} + \hat{j} * m_{R_M} + \hat{k} * n_{R_M}\right) \qquad (4.8)$$

where, the direction cosines of the vector $\vec{R}_M$ are $l_{R_M}, m_{R_M}, n_{R_M}$.

The instantaneous velocity vector of the target is $\vec{V}_T$, given by:

$$\vec{V}_T = V_T * \left(\hat{i} * l_{V_T} + \hat{j} * m_{V_T} + \hat{k} * n_{V_T}\right) \qquad (4.9)$$

where, the direction cosines of the vector $\vec{V}_T$ are $l_{V_T}, m_{V_T}, n_{V_T}$.

The instantaneous velocity vector of the weapon is $\vec{V}_M$, given by

$$\vec{V}_M = V_M * \left(\hat{i} * l_{V_M} + \hat{j} * m_{V_M} + \hat{k} * n_{V_M}\right) \qquad (4.10)$$

where, the direction cosines of the vector $\vec{V}_M$ are $l_{V_M}, m_{V_M}, n_{V_M}$.

4.3.1 Constraint on Beam Width

The sine of the angle subtended at the origin by the two position vectors, OT and OM is given by the vector cross product of the vectors $\vec{R}_M$ and $\vec{R}_T$

$$Sin\Theta = \frac{\left(\vec{R}_M \ X \ \vec{R}_T\right)}{R_M * R_T} \qquad (4.11)$$

Generally, the beam width of the observation device is selected such that, at no point of time during the flight of the weapon, does the magnitude of Θ exceeds half the beam width, as otherwise, the weapon would not be *seen* by the observation device. If on the other hand, an observation device with a *specified beam width* is constrained to be chosen for this system, then *the magnitude of its half beam width will constrain* the choice of the other subsystem parameters. It is necessary to ensure that, at no point during the flight of the weapon, does the magnitude of Θ exceed half the beam width of the chosen device.

4.3.2 Offset of Weapon from Line-of-Sight

The offset of the instantaneous position of the weapon from the LOS is needed in the guidance equation. This is obtained as

$$MP = R_M * \mathrm{Sin}\Theta \tag{4.12}$$

4.3.3 Rate of Change of Θ

In order to provide damping in the weave mode, the rate of change of the magnitude of the offset is used in Eqn (4.3). It is readily seen that the rate of change of MP is proportional to the rate of change of the included angle subtended at the origin by the two position vectors, OT and OM. However, since these vectors can have independent rotations about the origin, under the influence of their velocity vectors $\vec{V}_T$ *and* $\vec{V}_M$ respectively, vector differentiation needs to be used to get the magnitude of the rate-of-change of the included angle. This is obtained as follows:

Using the scalar dot product of the two position vectors, the cosine of the included angle is defined as

$$Cos\Theta = \frac{\left(\vec{R}_M \bullet \vec{R}_T\right)}{R_M * R_T} \tag{4.13}$$

Differentiating both sides with respect to time and rearranging, we get

$$\frac{d\Theta}{dt} = -\left(\frac{\vec{R}_T \bullet \vec{V}_M + \vec{V}_T \bullet \vec{R}_M}{R_T * R_M} - Cos\Theta * \left(\frac{\vec{R}_M \bullet \vec{V}_M}{R_M^2} + \frac{\vec{R}_T \bullet \vec{V}_T}{R_T^2}\right)\right) \div Sin\Theta \tag{4.14}$$

4.3.4 Weapon Manoeuvre Plane

While the command guidance policy demands lateral accelerations on the weapon whose magnitude is given by Eqn (4.3), the plane in which the weapon should manoeuvre under the influence of this lateral acceleration, should pass through the instantaneous position of the weapon and contain the instantaneous LOS vector. This requirement ensures that the weapon takes the shortest path towards the target. The unit normal vector to this manoeuvre plane is got by the vector cross product of OM and OT. Thus,

$$\hat{N} = \frac{(\bar{R}_M \; X \; \bar{R}_T)}{|R_M \; X \; R_T|} \tag{4.15}$$

4.3.5 Direction of Lateral Acceleration

Lateral acceleration can be generated on the weapon only in a pencil of planes containing its instantaneous velocity vector. Therefore, having selected the manoeuvring plane (defined by its unit normal vector in Eqn (4.15)) the direction of the lateral acceleration generated on the weapon is got by the vector cross product of this unit normal vector with the instantaneous velocity vector of the weapon. This ensures that while the lateral acceleration vector is normal to the instantaneous velocity vector, it also lies in the selected manoeuvre plane. Thus, the unit vector in the direction of the demanded lateral acceleration is given by

$$\hat{A}_{lateral} = \frac{(\hat{N} \; X \; \bar{V}_M)}{|\hat{N} \; X \; \bar{V}_M|} \tag{4.16}$$

4.3.6 Net Acceleration on Weapon

In addition to the lateral acceleration, the weapon is subjected to two other accelerations. These are: the longitudinal acceleration due to the thrust of its propulsion device and the acceleration due to gravity. While the acceleration due to its thrust is *assumed* to have the same direction as that of its instantaneous velocity vector, the direction of the acceleration due to gravity is directed parallel to the initially defined vertical axis, ZO. The net acceleration on the weapon is therefore the vector sum of these three accelerations.

$$\bar{A}_{net} = \hat{A} * A_{lateral} + \hat{V}_M * A_{thrust} * \hat{k} * g \tag{4.17}$$

4.3.7 Velocity and Position of Weapon

Till the next command update is received by the weapon, it will manoeuvre under the influence of the net acceleration. The velocity achieved by the weapon and its position at every interval of time can be got by successive integration of the net acceleration. Thus, the velocity of the weapon at any interval of time is given by

$$\vec{V}_{M_{(t+\delta t)}} = \vec{V}_{M_{(t)}} + \vec{A}_{net} * \delta t \tag{4.18}$$

$$\vec{R}_{M_{(t+\delta t)}} = \vec{R}_{M_{(t)}} + \frac{\vec{V}_{M_{(t+\delta t)}} + \vec{V}_{M_{(t)}}}{2} * \delta t \tag{4.19}$$

Using the above two equations, one can determine the trajectory of the weapon manoeuvring in three-dimensional space under the influence of the command guidance policy.

A three-dimensional computer program has been developed for use in simulation studies, based on the above set of trajectory equations. This programme is an adaptation of the two-dimensional programme listed by Zarchan[1]. During simulation studies, the system functional parameters are varied systematically to determine their influence on the final miss distance achieved by the weapon. The final set of essential system functional features required for it to meet a specified need is based on the assessment of their influences on the miss distance. An example to illustrate the methodology developed in this chapter is included in Appendix A.

In this example, the technique developed in this chapter is employed to identify the *kernel of system functional features,* which are essential for the design of an area defence weapon to counter attacks from low-flying aircraft. This weapon design is based on the use of the basic command guidance policy.

It is seen from the example illustrated (Appendix A) that the basic beam rider policy *does not lead to a direct hit* on the target. This is an inherent limitation of the basic command guidance policy. In the cited example, the minimum miss distance achievable is of the order of 13 m.

This large order of miss distance is due mainly to the basic beam rider policy making use of only the *current position information of the target,* for the guidance of the weapon. The policy demands lateral acceleration on the weapon that is proportional to the offset distance of the weapon from the *current LOS.* The information about the acceleration of the LOS under the influence of the velocity of the target is not made use of for the guidance of the weapon (Lateral acceleration proportional to the rate of change of the current offset distance, which is demanded by the modified guidance policy as per Eqn (4.3) serves only to damp out the weave mode oscillations).

Thus, under the basic command guidance policy, the guidance scheme does not *anticipate* the acceleration of the LOS. Therefore, the weapon always lags behind the target resulting in a finite miss distance.

If additional information about the acceleration of the target LOS is made use of in the guidance policy, a greater amount of *anticipation can be built into the guidance of the weapon.* This would result in the achievement of closer miss distances. CLOS policy makes use of such a feature. Details of this policy are developed below.

4.4 COMMAND TO LINE-OF-SIGHT POLICY (CLOS)

The command to line-of-sight (CLOS) guidance policy is a modification of the basic command guidance policy where, information about the acceleration of the LOS is additionally made use of while demanding lateral acceleration on the weapon.

Since the observation device is slaved to the target, the target LOS rotates in three-dimensional space under the influence of the velocity of the target. The rotational velocity vector of the LOS is obtained by taking the vector cross product of the LOS vector and the instantaneous velocity vector of the target. Thus,

$$\bar{\omega}_{los} = \frac{\left(\bar{R}_T \; X \; \bar{V}_T\right)}{R_T^2} \tag{4.19}$$

Using vector differentiation, the rotational acceleration of the target LOS can be obtained as

$$\frac{d\bar{\omega}_{los}}{dt} = \frac{\left(\bar{R}_T \; X \; \bar{A}_T\right)}{R_T^2} - 2 * \left(\bar{\omega}_{los} * \frac{\bar{R}_T \bullet \bar{V}_T}{R_T^2}\right) \tag{4.20}$$

where $\bar{A}_T$ is the acceleration vector of the target.

The acceleration vector, perpendicular to the LOS, is given by the vector cross product

$$\bar{A}_{T_{perp}} = \frac{\bar{R}_T \; X \; \bar{A}_T}{R_T} \tag{4.21}$$

From Eqns (4.20) and (4.21), the acceleration vector of the target perpendicular to the LOS is

$$\vec{A}_{T_{perp}} = R_T * \left(\frac{d\vec{\omega}_{los}}{dt} \right) + 2 * \vec{\omega}_{los} * \left(\frac{dR_T}{dt} \right) \quad (4.22)$$

In CLOS policy, the angular velocity and the angular acceleration of the position vector of the weapon are sought to be made identical to those of the LOS vector.

This is achieved *by demanding additional acceleration* on the weapon, perpendicular to its instantaneous position vector, the magnitude of which is given by Eqn (4.23).

$$\vec{A}_{M_{perp}} = R_M * \left(\frac{d\vec{\omega}_{los}}{dt} \right) + 2 * \vec{\omega}_{los} * \left(\frac{dR_M}{dt} \right) \quad (4.23)$$

This modified guidance policy, as is to be expected, results in the achievement of closer miss distance than the basic beam rider policy. The schematic diagram of the modified CLOS policy is shown in Fig. 4-5.

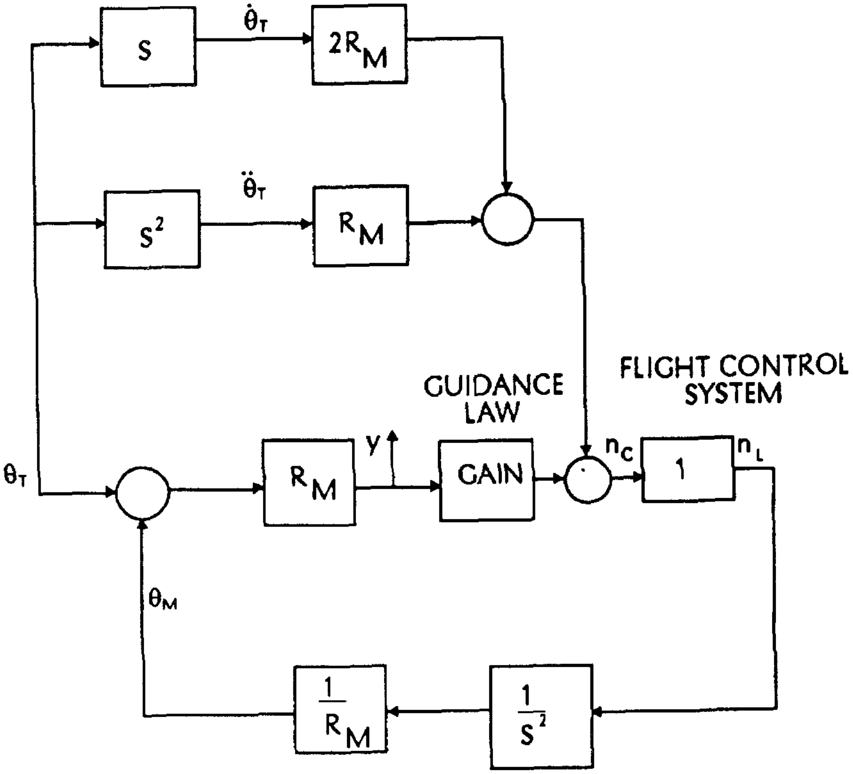

Figure 4-5. Command to line-of-sight guidance system

To illustrate the advantages of the CLOS policy over the basic beam rider policy, computer simulation studies have been repeated for the Area Defence Weapon detailed in Appendix A. The results of the CLOS policy are presented in Appendix B.

As expected, this modified policy leads to much closer miss distances than the basic beam rider policy. It is seen from Fig. A-7 that the basic beam rider policy could achieve a miss distance of 18.5m only at a speed ratio of 1.5 and at the guidance gain of $10 m/s^2/m$. However, from Fig. B-6, it is seen that under the same conditions, the modified policy could achieve a miss distance of just 1.1m. *The maximum demanded lateral acceleration is practically identical for both the guidance policies. However, the miss distance achieved by using the CLOS policy is an order of magnitude lower than what can be achieved by the basic beam rider policy.*

Jane's Weapon Systems lists the *Rapier* missile system following the command guidance policy as a *Hittile* which is the lightest missile of its class and it reportedly scores a direct hit with the target. Consequently, its warhead weight is the least. Perhaps, the RAPIER missile system follows the CLOS guidance policy, which is only a modified form of the basic command guidance policy. Only by using such a varients of the command guidance policy can one expect to achieve such close miss distances.

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CHAPTER 5

SYSTEMS STUDY FOR PROPORTIONAL NAVIGATION WEAPON SYSTEMS

Another guidance policy that finds wide application in guided weapon systems is the *proportional navigation policy* which almost all *homing missiles* follow. The *constant bearing course* is one of the earliest variant of this form of guidance policy. In this variant of the proportional navigation policy, the weapon is launched giving it a lead in the anticipated direction of movement of the target. If the lead angle was correct, in the time taken for the weapon to reach the target, the target would have moved *just the right amount*, to result in an interception with the weapon. In such an event, the trajectory of the weapon would be a straight line, thus requiring *no rate of change to its flight path.* As a consequence, no manoeuvring force needs to be generated on the weapon. However, the flight path of the weapon would need corrections to its straight line path, when the prediction of the initial lead angle was either incorrect, or when the target resorted to an evasive manoeuvre after the launching of the weapon against it. In such cases, the proportional navigational policy demands a *rate of change to its flight path which is proportional to the rate of change of the sight line.*

In this Chapter, it is proposed to develop a methodology for identifying the system characteristics needed for the design of such a weapon following the proportional navigation guidance policy. An understanding of the weapon system from the point of view of the systems designer is necessary.

5.1 PROPORTIONAL NAVIGATION COURSE—SYSTEMS DESIGNER'S POINT OF VIEW

The systems designer views the trajectory of the weapon following the proportional navigation policy, essentially as the results of the mutual interaction between two points. The first point representing the target moves in three-dimensional space under the commands of the adversary. The second point, representing the weapon moves in three-dimensional space under the action of the proportional navigation policy.

The weapon system has a homing head (usually a radar device or an infra-red device) located in its forebody. The beam of the homing head is slaved to the target by a servomechanism, thus establishing a LOS. The servomechanism keeps the beam locked-on to the target. Therefore, as the target manoeuvres in three-dimensional space, the LOS also rotates under the influence of the relative velocity of the target, as seen from the weapon. The rate-of-rotation of the guidance beam is a measure of the rate-of-rotation of the LOS. This perception of the systems designer is the basis for the development of a system model for the weapon following a proportional navigational guidance policy. The typical scenario of an encounter between a target and a weapon following a proportional navigational guidance policy is shown in Fig. 5-1. An inertial frame of reference is selected to describe the motions of the target and the weapon. *O* is the origin of the inertial frame of reference, and the points *T* and *M* represent the locations of the target and the weapon, respectively. Similaly, V_T and V_M are the velocity vectors of the target and the weapon, respectively.

5.1.1 Mathematical Model

The instantaneous position vector of the target is OT. This is expressed vectorially by the equation

$$\vec{R}_T = R_T * (\hat{i} * l_{R_T} + \hat{j} * m_{R_T} + \hat{k} * n_{R_T}) \tag{5.1}$$

where, the direction cosines of the position vector $\vec{R}_T$, are

$l_{R_T}, m_{R_T}, \text{ and } n_{R_T}$

The instantaneous position vector of the weapon is OM. This is expressed vectorially by the equation:

$$\vec{R}_M = R_M * (\hat{i} * l_{R_M} + \hat{j} * m_{R_M} + \hat{k} * n_{R_M}) \tag{5.2}$$

where, the direction cosines of the position vector $\vec{R}_M$, are

$l_{R_M}, m_{R_M}, \text{ and } n_{R_M}$

The LOS vector can be got from the above two position vectors as

$$\overline{T\vec{M}} = \vec{R}_T - \vec{R}_M \tag{5.3}$$

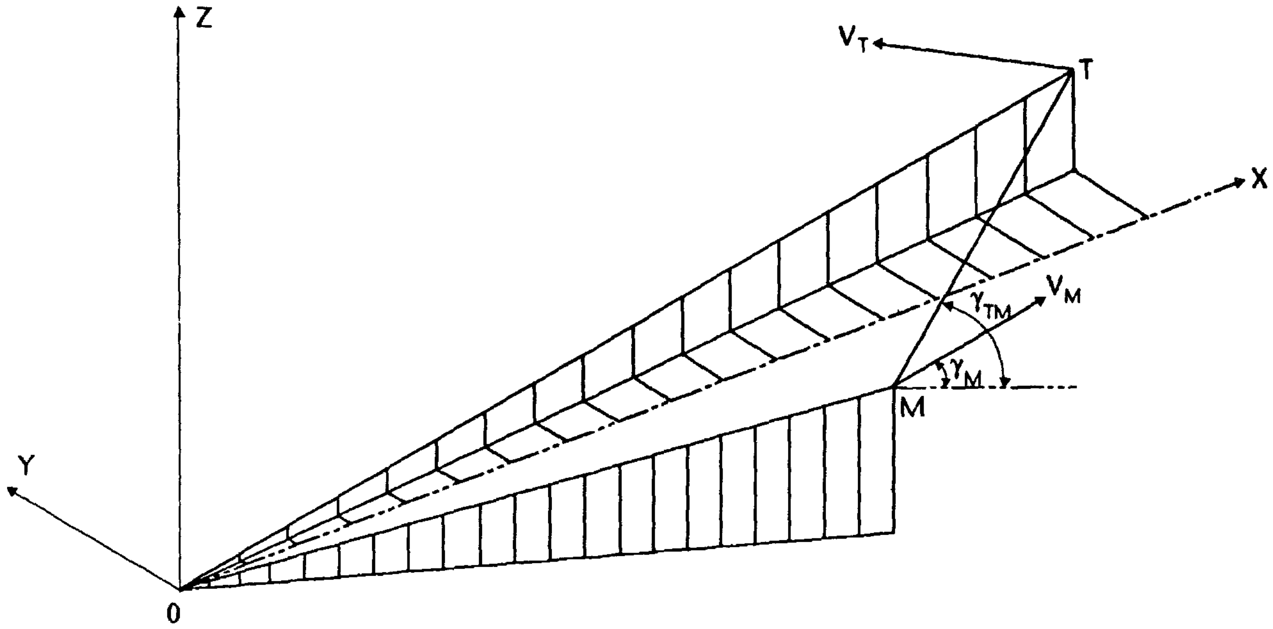


Figure 5-1. Scenario of target and weapon following proportional navigational policy

The velocity vectors $\vec{V}_T$ *and* $\vec{V}_M$ of the target and the weapon can be expressed in terms of their direction cosines as follows:

$$\vec{V}_T = V_T * (\hat{i} * l_{V_T} + \hat{j} * m_{V_T} + \hat{k} * n_{V_T}) \tag{5.4}$$

$$\vec{V}_M = V_M * (\hat{i} * l_{V_M} + \hat{j} * m_{V_M} + \hat{k} * n_{V_M}) \tag{5.5}$$

The relative velocity of the target, as seen from the weapon, can be expressed as

$$\vec{V}_{rel} = \vec{V}_T - \vec{V}_M \tag{5.6}$$

The LOS vector $\overline{T\vec{M}}$ rotates in space under the influence of the relative velocity of the target with respect to the weapon. The angular rate of rotation of the LOS is given by the vector cross product of the LOS vector and the relative velocity vector. Thus,

$$\frac{d}{dt}(\gamma_{TM}) = \frac{\overline{T\vec{M}} \ X \ \vec{V}_{rel}}{TM^2} \tag{5.7}$$

The proportional navigational policy can now be expressed as

$$\frac{d}{dt}(\gamma_M) = k * \frac{d}{dt}(\gamma_{TM}) \tag{5.8}$$

where,

$\frac{d}{dt}(\gamma_M)$ is the angular rate of rotation of the flight path of the weapon, and k is the constant of proportionality, also known as the *guidance gain.*

As a consequence of adopting this guidance policy, the lateral accelerations that is demanded on the weapon for its manoeuvre has magnitude which is given by the relation

$$A_{lateral} = k * \frac{d}{dt}(\gamma_M) * V_{close} \tag{5.9}$$

where,

V_{close} is the closing velocity of the weapon towards the target.

The closing velocity of the weapon towards the target is given by the scalar dot product of the LOS vector with the relative velocity vector. Thus,

$$V_{close} = -\left(\frac{\vec{TM} \bullet \vec{V}_{rel}}{TM} \right) \tag{5.10}$$

The cosine of the angle subtended by the LOS vector with the velocity vector of the weapon is given by the scalar dot product of the LOS vector with the velocity vector of the weapon. Thus,

$$\cos(\theta) = \frac{\vec{TM} \bullet \vec{V}_M}{TM * V_M} \tag{5.11}$$

Generally, the angle θ should be less than the look-angle of the detector located in the homing head. If θ were greater than the look-angle of the detector, then the target would not be detected by the weapon. The objective of the systems study therefore is to so select the parameters defining the weapon system in such a manner that under no condition of its flight should the detector loose sight of the target.

5.1.2 Weapon Manoeuvre Plane

The *magnitude* of the lateral acceleration demanded by the proportional navigation guidance policy is given by Eqn (5.2). The *direction* of the lateral acceleration should be such that, under its influence, the weapon should head towards the target. To ensure this requirement, the weapon should manoeuvre in a plane that contains the LOS vector and the relative velocity vector. The unit normal vector defining such a manoeuvre plane is obtained by the vector cross product of the unit vector along the LOS and the unit vector along the relative velocity vector. Thus,

$$\vec{N} = \frac{\vec{TM} \, X \, \vec{V}_{rel}}{\left| \vec{TM} \, X \, \vec{V}_{rel} \right|} \tag{5.12}$$

In this manoeuvre plane, the configuration of the weapon should generate the lateral accelerations demanded by the guidance policy. Since the configuration can generate lateral aerodynamic lift forces needed for manoeuvering, only in a pencil of planes containing its instantaneous velocity vector, the required weapon manouevring plane should be identified with one of the planes belonging to this pencil of planes. The unit vector in the direction of the lateral acceleration generated on the weapon is obtained by the vector cross product of the unit normal vector defined by the Eqn (5.12) and the unit vector in the direction of the instantaneous velocity vector of the weapon. Thus,

$$\hat{A}_{lateral} = \frac{\hat{N} X \vec{V}_M}{|\hat{N} X \vec{V}_M|} \tag{5.13}$$

5.1.3 Net Acceleration on Weapon

The net acceleration on the weapon is the sum of three accelerations. These are:

- The lateral acceleration demanded on it by the guidance policy which is directed along $\hat{A}_{lateral,}$
- The longitudinal acceleration imparted to it by the propulsion devices which *is assumed* to be directed along the instantaneous velocity of the weapon, V_M and
- The acceleration due to gravity directed along $\hat{k}$.

Thus,

$$\vec{A}_{net} = \hat{A}_{lateral} * A_{lateral} + \hat{V}_M * A_{longitudinal} + \hat{k} * g \tag{5.14}$$

5.1.4 Trajectory of Weapon

The weapon will manoeuvre under this net acceleration till the next command update is received by it. The velocity achieved by the weapon during this period and the distance covered by it in this period of time are given by following vector equations

$$\vec{V}_{M_{t+\delta t}} = \vec{V}_{M_t} + \vec{A}_{net} * \delta t \tag{5.15}$$

$$\vec{R}_{M_{t+\delta t}} = \vec{R}_{M_t} + \frac{\vec{V}_{M_{t+\delta t}} + \vec{V}_{M_t}}{2} * \delta t \tag{5.16}$$

Using the above two equations, one can determine the trajectory of the weapon manoeuvring in three-dimensional space under the influence of the proportional navigation policy.

A three-dimensional computer program has been developed for use in simulation studies, based on the above set of trajectory equations. This programme is an adaptation of the two-dimensional programme listed by Zarchan[1]. During simulation studies, the system functional parameters are varied systematically to determine their influence on the final miss distance. The final set of essential system functional features needed for the design of a weapon to meet the user's stated need is based on the assessment of their influences on the miss distance. The set of compatible parameters that results in the achievement of minimum miss distance is selected for the detailed design of the weapon. An example to illustrate the methodology developed in this chapter is included in Appendix C.

In this example, the technique developed in this Chapter is employed to identify the *kernel of system functional features*, which are essential for the design of a *shoulder fired weapon* to counter attacks from low-flying aircraft following the proportional navigational guidance policy.

Based on an analysis of the results of the computer simulation studies, the thrust policy for the system could either be a boost-sustained policy or a boost-coast policy. The set of compatible essential system functional features needed for the design of a shoulder fired weapon system to meet the user's need can be specified as:

Case 1. Boost-sustain Policy

The guidance gain	≥ 2.5
The rise time of the weapon in its short period mode	$= 40\ ms$
The frequency at which the commands need to be updated	$\leq 0.1\ s$,
The longitudinal acceleration needed for launching the weapon	$\geq 200\ m/s^2$
$2.5s. \leq$ *boosting time*	≤ 3.0 s.
Longitudinal acceleration in the sustained phase of flight	$= 0.0$
Maintain boost-end speed till interception	
Maximum lateral acceleration demanded on the weapon	$39.28\ m/s^2$

Case 2. Boost-coast Policy

The longitudinal acceleration needed for launching the weapon	$\geq 200\ m/s^2$
$2.5\ s \leq$ *boosting time* ≤ 3.0 s.	
Coast the weapon till interception	
Boost end value	$\mu \geq 6000$

• The maximum lateral acceleration demanded on the weapon	$39.28\ m/s^2$
• Miss distance	$\leq 1\ m$

REFERENCE

1. Zarchan, P. Fundamentals of tactical missile guidance. *Progress in Aeronautics and Astronautics.* AIAA Vol. 124, 1990.

CHAPTER 6

AERODYNAMIC CONFIGURATION DESIGN

The technique of systems study was applied in the earlier chapters to assess the miss distance resulting from the interaction of the chosen guidance policy with the dynamics of the weapon configuration. The *outer loop* of the weapon system deals with this interaction between the guidance policy selected for the system and the short period dynamics of the configuration. The effect of this interaction on the miss distance achieved by the weapon was analysed. Systems study enabled us to find the *kernel of system functional features* of the outer loop that was essential to achieve acceptable miss distance of the weapon from the target. Two of the five systems functional features that have been identified by such systems study are relevant for the detailed design of the configuration. These are:

- The lateral acceleration demanded by the chosen guidance policy to define its trajectory, and
- The lag that can be tolerated by the system, represented by the rise time, t_{rise}.

A method is developed in this Chapter for the determination of the essential functional characteristics of the configuration that are needed to meet these two system level demands.

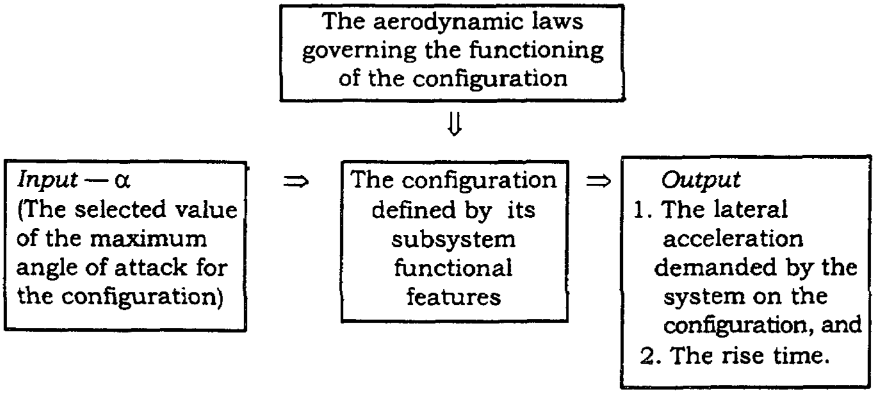
The design of the aerodynamic configuration is concerned with the determination of the external aerodynamic shape of the flying weapon. The aerodynamic configuration serves the above two important system functions, besides providing the volume needed for accommodating all the airborne subsystems.

Various configuration shapes can be thought of for fulfilling these functions. Some of them could be canard-controlled configurations, some wing-controlled configurations while yet others tail-controlled configurations. Further, the configurations could have different types of forebodies like hemispherical noses, conical or ogival noses. Aerodynamic configuration design involves the determination of all these features.

How does one go about the task of determining the external aerodynamic shape of the configuration? Is there only one unique shape for meeting these requirements on the configuration, or are there other configuration possibilities? If more than one configuration possibility exists, how can one identify all these configurations?

Systems study comes to our rescue once again to find answers to these questions. The techniques of systems study, however, are now applied at the subsystem level, *in a recursive fashion*, to find the *kernel of essential configuration features* that are necessary *to meet the two system level demands on the configuration*. According to Dixon[1], there are four specific elements needed for conducting such systems study (Table 6-1).

Table 6-1. Elements of system study



The configuration is defined by a minimum set of its subsystem functional features that are adequate to describe its functioning *in an aggregate sense*. It can be seen that the systems designer needs a model for describing the functioning of the subsystem. For defining the model, the systems designer takes a holistic view of the subsystem instead of going into the details of the subsystem. An understanding of the perception of the systems designer about the configuration of the weapon is necessary.

6.1 SYSTEMS DESIGNER'S POINT OF VIEW

The configuration is viewed by the configuration sub-systems designer essentially as a long cylinder, non-uniform both in its mass and stiffness distribution. This cylinder is set at an angle of attack, α with respect to the prevailing wind-stream, for generating the *system level demand* for lateral lift force needed for manoeuvring. The aerodynamic lift, L_{conf} so generated on the configuration in the required lateral direction, is considered to be contributed, essentially, by three lifting elements located at specific distances from the centre of gravity of the configuration. These three locations are identified, subsequently, with the locations of the centres of action of the nose lift, the wing lift and the tail lift, respectively. This idealisation of the configuration is most general since it is always possible to identify, either all three, or at least two, out of the following three lifting elements in every configuration:

- The nose lift L_n, which consists of the lift of the tapered forebody.
- The wing lift L_w*.
- The tail lift L_t,

One of these three lifting elements can also be selected for control purposes, when the selected lifting element has the additional capability for independent deflection. Depending upon which of these three lifts producing elements is selected for control purposes, one gets a canard-controlled configuration, a wing-controlled configuration or a tail-controlled configuration. This idealisation of the configuration thus has all the functional features that are essential and adequate for describing the functioning of a configuration. With this idealisation, the processes of systems study now involves the distribution of L_{conf} on these three lifting elements and selecting their locations appropriately for assuring the required rise time t_{rise}.

6.2 CONFIGURATION MODEL

The model for the configuration, depicting the above perceptions of the systems designer, is shown in Fig. 6-1. The three lifting elements are indicated in the figure by

- The nose lift L_n located at a distance of x_n from the nose of the configuration.
- The wing lift L_w, located at a distance of x_w from the nose of the configuration.

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* For want of an alternate term, the term 'wing lift' is used here to denote the lift produced by middle lifting element, out of the three lifting elements of the idealised configuration. This term, however, should not be taken as implying the principal lifting element of the configuration.

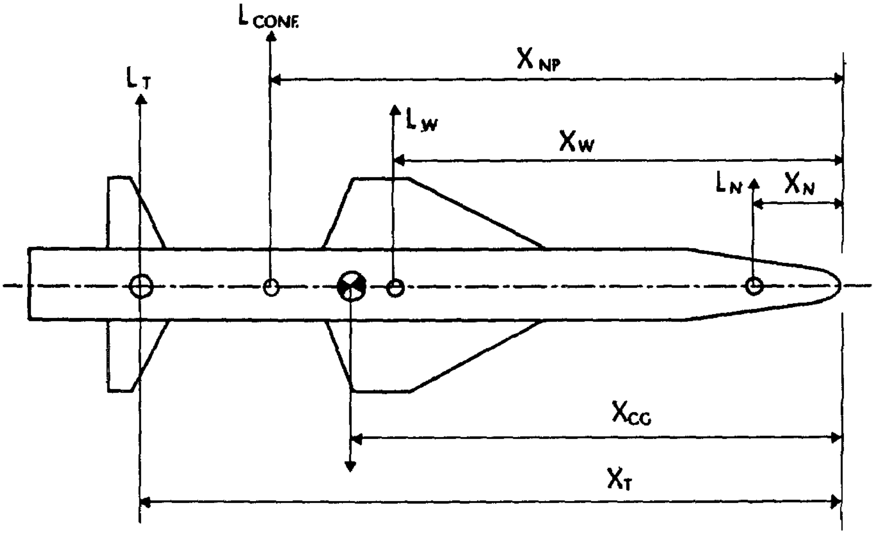

Figure 6-1. System model for the configuration

- The tail lift L_t located at a distance of x_t from the nose of the configuration.
- The configuration lift L_{conf} is located at the neutral point of the configuration, which is at a distance of x_{np} from the nose of the configuration.

How does the *first system level demand* on the configuration, namely that of the generation of the aerodynamic manoeuvring lift force, answered by this model ?

6.3 GENERATION OF CONFIGURATION LIFT FORCE

The lift force L_{conf} generated on the configuration when it is set at an angle of attack α acts at the neutral point of the configuration, which is located at a distance of x_{np} from the nose of the configuration. The relations between the configuration lift force and its constituents, located on the three lifting elements, are represented by the pair of equations

$$L_{conf} = L_n + L_w + L_t \tag{6.1}$$

$$\left(L_{conf} * x_{np}\right) = \left(L_n * x_n\right) + \left(L_w * x_w\right) + \left(L_t * x_t\right) \tag{6.2}$$

The location of the neutral point on the configuration is thus seen to be controlled by the lift distribution function and by the selection of the locations of the centres of action of the three lifting elements. One can find several possible locations for the neutral point, satisfying the above two equations. *Is there any preferred location for the neutral point, out of all such solutions?* Some of the considerations for the location of the neutral point are given below.

6.3.1 Requirement of Static Stability

Static stability deals with the behaviour of the configuration, when it is disturbed from its equilibrium state. If the configuration has a *tendency* to restore itself to its original equilibrium state automatically whenever it is disturbed from it, then the system is said to be statically stable. This implies that the act of taking the configuration away from its equilibrium state should set up corrective forces automatically to bring it back to its equilibrium state. In statically stable configurations, *the necessary condition for assuring static stability is that the neutral point should be located aft of the location of its centre of gravity.* Thus,

$$x_{np} \geq x_{cg} \tag{6.3}$$

6.3.2 Static Margin

The location of the neutral point, aft of the location of the centre of gravity, is a necessary condition for assuring static stability. The distance between the neutral point and the centre of gravity of the configuration is known as the *static margin,* usually denoted by the letter *h.* The static margin is given by the relation

$$h = \left(x_{np} - x_{cg}\right) \tag{6.4}$$

The static margin is higher when the neutral point is located more aft from the centre of gravity. Different values of the static margin can be obtained by appropriate location of the neutral point. The only stipulation is that the value of the static margin so obtained should be positive to assure static stability.

What should be its magnitude? *Is there any preferred value for the static margin?* To answer this question, aspect of the controllability of the configuration needs to be addressed.

6.3.3 Controllability

In order to set the configuration at the angle of attack α, the selected control element needs to be deflected in the appropriate direction through an angle . The centre of action of the control lift force is at a distance of x_c from the nose of the configuration. When the configuration is thus *trimmed* and set at the required angle of attack α, *the net aerodynamic moment about its centre of gravity is zero.* This requirement is expressed by the equation

$$L_{conf} * \left(x_{np} - x_{cg}\right) + L_c * \left(x_c - x_{cg}\right) = 0 \tag{6.5}$$

where,

$\left(x_{np} - x_{cg}\right)$ is the now familiar static margin h, and

$\left(x_c - x_{cg}\right)$ is known as the control arm l_c

The control force needed to trim the configuration at the required angle of attack can be obtained from Eqn (6.5) as:

$$L_c = - L_{conf} * \left(\frac{h}{l_c}\right) \tag{6.6}$$

It is noticed that the magnitude of the control force needed for generating the required configuration lift force is uniquely determined by the ratio (h/l_c). The inverse of this ratio, (l_c/h) is known as the *gearing ratio*; the higher the gearing ratio, the lower is the control force required to trim the configuration at the required angle of attack. *The availability of the control power for actuating the selected control element will therefore constrain the specification of the gearing ratio.* Indirectly, this specification of the gearing ratio will constrain the location of the neutral point. Selecting the gearing ratio, so as to minimise the control power requirement is, therefore, a prerequisite for efficient designs.

6.3.4 Manoeuvre Force

Under *trim condition, the net manoeuvring lift force on the* configuration is given by the summation of the lift due to angle of attack and the lift due to control deflection. Thus,

$$L_{manoeuvre} = L_{conf} + L_c$$

l_c can be expressed in terms of L_{conf}, using the trim relation given by Eqn (6.6). The net manoeuvring force can then be expressed as

$$L_{manoeuvre} = L_{conf} * \left(1 - \frac{h}{l_c}\right) \tag{6.7}$$

It is noticed that the configuration lift force is modified by the factor $\left(1 - \frac{h}{l_c}\right)$ to get the manoeuvre force. For positive values of the gearing ratio, as *in tail-controlled configurations, the manoeuvre force is always less than the configuration lift force.* In efficient tail-controlled configuration designs, *the gearing ratio is selected as large as possible to minimise the inevitable loss to the manoeuvre force.* We concluded earlier that the gearing ratio needs to be properly selected to have good controllability. The selection of the gearing ratio should therefore be made judiciously so that the configuration is controllable, yet does not result in any serious diminution to the net manoeuvre force.

The *first systems demand on the configuration,* namely that of generating the required manoeuvre aerodynamic force is thus, intimately connected with the location of the neutral point on the configuration.

6.4 ENSURING NECESSARY SYSTEM LAG ON CONFIGURATION

The *second system level demand* on the configuration, namely that of ensuring the acceptable range in the value of the system lag has been obtained by conducting systems study on the outer loop of the weapon system. It is specified by the time parameter t_{rise}, which is defined as the time taken by the configuration to generate the demanded lateral acceleration, from the instant the demand for such lateral acceleration is received.

How is the lateral acceleration actually generated on the configuration? The fastest servomechanism in the entire configuration is that of the control subsystem. It can be justifiably *assumed,* that the guidance demand for lateral acceleration to have been executed instantaneously by the control subsystem. This results in the generation of a step control force, L_c. The response of the configuration to this step control force is the dynamic response of the configuration, a consideration which is necessary for understanding the lag of the system.

6.4.1 Short Period Dynamics of Configuration

On the generation of the step control force, the configuration is subjected to a step control moment of magnitude $(L_c * l_c)$. Under the influence of this moment, the configuration will start rotating about its centre of gravity in the direction of the applied control moment. As the configuration starts rotating, the angle of attack of the configuration is altered from its previous equilibrium value. This alteration in turn generates an additional lateral lift force on the configuration, acting through the neutral point of the configuration. This additional lateral force causes a moment about the centre of gravity of the configuration. The magnitude of the additional moment is (L_{conf} *h). The resultant behaviour of the configuration under the influence of these two moments is the subject of study of the short period dynamics of the configuration.

Whenever the configuration is disturbed from its equilibrium flight condition, the manner in which it is restored to its original flight condition is the subject of study of the short period dynamics of the configuration. The prequisite for analysing the short period dynamics of the system is that the *system should be statically stable.* As seen earlier, this requirement is obtained by the distribution of the lateral lift on the selected three lifting elements so as to assure a positive static margin. In such statically stable configurations, the aerodynamic moment generated on it (as the angle of attack is altered), is opposite in direction to the applied control moment. Under the influence of these two opposing moments, the configuration would undergo *weather cock* or *short period oscillations*. The typical response of the configuration to a step control input is illustrated in Fig. 6-2. The angle of attack of the configuration while rising to its demanded value would overshoot and go past the demanded value. It would then execute a series of damped weather-cock oscillations before settling down to the demanded value. The lateral acceleration demanded on the configuration would also keep pace with variations in the angle of attack and follow a similar oscillatory trend. As a consequence, the configuration would take a finite time to generate the lateral acceleration demanded on it by the guidance policy. The time taken for the lateral acceleration to reach the demanded value, *the first occasion*, is termed the rise time of the configuration.

By identifying the system lag with the rise time of the configuration in the short period mode, the dynamic *short period characteristics* of the configuration can be tailored to ensure the acceptable levels of the systems lag. However, to accomplish this task, the parameters defining the configuration, needs to be linked with the system time lag, t_{rise} .

6.4.2 Relating t_{rise} with Configuration Parameters

Considering the configuration as a second order system, the rise time can be related to the frequency and damping of the configuration in its short period mode. The rise time can be related to the damped short period frequency ω_d and its damping factor ζ_{sp} as follows[2]:

Defining the rise time of the system as the time required for the system to reach the demanded value of the lateral acceleration, *on the first occasion,*

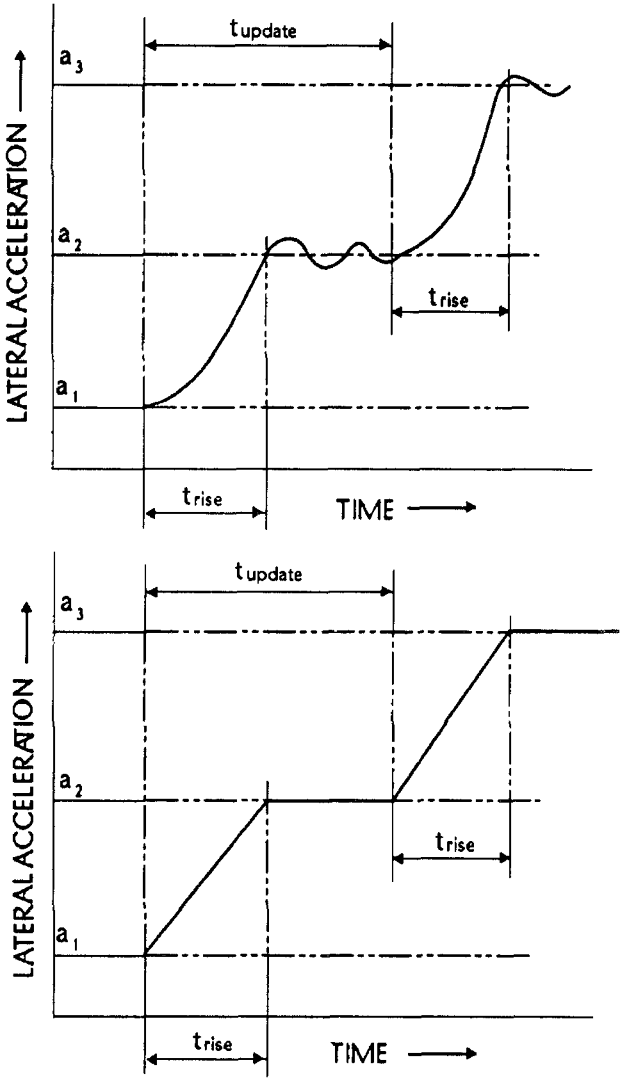

Figure 6-2. Typical step response of the configuration (a) Lag in demanded acceleration, (b) Idealisation of missile response lag

$$t_{rise} = \frac{\Pi - \beta}{\omega_d} \tag{6.8}$$

where,

$$\beta = \tan^{-1}\left(\frac{\sqrt{1 - \zeta_{sp}}}{\zeta_{sp}^{2}}\right) \tag{6.9}$$

ω_d, is the damped short period frequency given by

$$\omega_d = \omega_{sp} * \sqrt{1 - \zeta_{sp}^2} \tag{6.10}$$

ω_{sp}, is the undamped frequency in the short period mode,

ζ_{sp} is the damping factor in the short period mode.

Using these relations we can identify pairs of values of ω_{sp} and ζ_{sp} to meet the specification on t_{rise}. Table 6-2 lists pairs of values of ω_{sp} and ζ_{sp} for a range of values of t_{rise} from 0.01s to 0.10 s.

Of these pairs of parameters governing the rise time in guided weapons, the requirement of damping is very difficult to obtain solely by aerodynamic means. Usually the required amount of damping is achieved in guided weapons *by synthetic means* using rate-feedback in the control loop. The value of ζ_{sp} is therefore not a significant parameter influencing the aerodynamic design of guided weapon configurations. This leaves only ω_{sp} to link the specification of t_{rise} with configuration design.

From the aerodynamic theory of configurations, we know that the undamped short period frequency of any given configuration is related to its static margin and on the slope of its configuration lift-curve. This relationship is given by

$$\omega_{sp} = \sqrt{\left(\frac{q * S * Cl_{\alpha_{conf}} * h}{I}\right)} \tag{6.11}$$

where,

q, is the dynamic pressure, given by

$$q = \frac{1}{2} * \rho * V^2 * S$$

$Cl_{\alpha_{conf}}$ is the slope of the configuration lift curve,

I is the rotational moment of inertia of the configuration about a lateral axis passing through its centre-of-gravity,

S is the reference area used for normalising the lift force and

h is the static margin.

Table 6-2. t_{rise} **as function of** ω_{sp} **, and** ζ_{sp}

ζ_{sp}	$\beta = \tan^{-1}\frac{\sqrt{1-\zeta_{sp}^2}}{\zeta_{sp}}$	ω_{sp}	$\omega_d = \omega_{sp} * \sqrt{1-\zeta_{sp}^2}$	$t_{resp} = \frac{\pi - \beta}{\omega_d}$
(1)	*(2)*	*(3)*	*(4)*	*(5)*
0.1	1.4706289	167.920	167.078280	0.01000
		83.96	83.539142	0.02000
		55.973	55.692721	0.03000
		41.980	41.769571	0.04000
		33.584	33.415656	0.05000
		27.987	27.846380	0.06000
		23.989	23.868326	0.07000
		20.990	20.884785	0.08000
		18.658	18.564253	0.09000
		16.792	16.707828	0.10001
0.2	1.3694384	180.860	177.205860	0.01000
		90.430	88.602934	0.02000
		60.287	59.068622	0.03000
		45.215	44.301467	0.04000
		36.172	35.441173	0.05000
		30.143	29.534311	0.06000
		25.837	25.315124	0.07000
		22.608	22.150733	0.08000
		20.096	19.689540	0.09000
		18.086	17.720586	0.10000
0.3	1.2661036	196.600	187.544440	0.01000
		98.300	93.772223	0.02000
		65.533	62.514815	0.03000
		49.150	46.886111	0.04000
		39.320	37.508889	0.05000
		32.767	31.257407	0.06000
		28.086	26.792063	0.07000
		24.575	23.443055	0.08000
		21.844	20.838271	0.09000
		19.660	18.754444	0.10000
0.4	1.1592794	216.280	198.223880	0.01000
		108.140	99.111942	0.02000
		72.093	66.074628	0.03000
		54.070	49.555971	0.04000
		43.256	39.644777	0.05000
		36.047	33.037314	0.06000
		30.897	28.317698	0.07000
		27.035	24.777985	0.08000
		24.031	22.024876	0.09000
		21.628	19.822388	0.10000

In guided weapon designs, the reference area, S is taken as the cross-sectional area of the cylindrical portion of the configuration, usually, based on the maximum diameter, d of the cylindrical portion of the configuration. The maximum diameter is also used as the reference length, for normalising all length parameters.

The *second system level specification* on t_{rise} can be linked with the frequency ω_{sp} by the use Eqns (6.8), (6.9) and (6.10).

Likewise, Eqn (6.11) has enabled us to link, once again, ω_{sp} with the static margin of the configuration.

Both the systems level demands have thus, been related with the distribution of the configuration lift on three lifting elements and choosing their locations appropriately to assure stability and controllability requirements. This is the crux of configuration design. The different configuration possibilities for the design problem have now to be identified. Systems study technique is used once again for this task.

6.5 CONFIGURATION SYSTEMS STUDY

Equations (6.1) and (6.2) are the relevant relations that link the two system level demands with the three lifting elements and their locations on the configuration. While Eqn (6.1) relates the lateral force requirement with its distribution on the three lifting elements, Eqn (6.2) assures that their locations on the configuration are compatible with the requirements for stability and controllability.

6.5.1 Selection of Operating Angle of Attack

The lateral force needed on the configuration (to stay on the trajectory demanded by the chosen guidance policy) has been arrived from systems study of the outer loop. The required lateral force is generated on the configuration by setting the configuration at the required angle of attack. At this stage, the systems designer has to *select the maximum angle of attack for the configuration.* The limit on the maximum angle of attack of the configuration is constrained on the following three considerations:

(a) The stalling angle for the configuration, α_{stall}.

(b) The degree of nonlinear behaviour of the configuration lift with angle of attack, which can be conveniently handled in the design of the configuration.

(c) In some cases, such as when the designer wishes to use a Ramjet propulsion device in the design, the proper functioning of its ram-air intake would dictate the need for low angles of attack of the configuration.

Assuming a linear behaviour for the configuration in generating lift, the slope of the configuration lift curve can be determined once the maximum angle of attack is selected. The slope of the configuration lift curve is given by the relation

$$Cl_{\alpha_{conf}} = \left(\frac{\text{maximum demanded manoeuvre force}}{\frac{1}{2} * \rho * V^2 * S * \alpha_{conf_{\max}}} \right) \tag{6.12}$$

The lateral aerodynamic manoeuvre force is contributed by the three lifting elements. Therefore, following Eqn (6.1) the configuration lift force can be expressed in terms of its components as follows:

$$q * S * Cl_{\alpha_{conf}} * \alpha_{conf} = q * S * \left(Cl_{\alpha_{nose}} + Cl_{\alpha_{wing}} + Cl_{\alpha_{tail}} \right) * \alpha_{conf} \tag{6.13}$$

6.6 NORMALISED GOVERNING EQUATIONS

Using $Cl_{\alpha_{conf}}$ *as the normalising parameter,* we obtain the slopes of the lift curves on the three lifting elements as a proportion of the slope of the configuration lift curve. Thus,

$$\bar{a}_n + \bar{a}_w + \bar{a}_t = 1 \tag{6.14}$$

$$\begin{pmatrix} \bar{a}_n \geq 0.0 \\ \bar{a}_w \geq 0.0 \\ \bar{a}_t \geq 0.0 \end{pmatrix} \tag{6.14a}$$

where,

$\bar{a}_n = \left(\frac{Cl_{\alpha_{nose}}}{Cl_{\alpha_{conf}}} \right)$, the proportion of the slope of the nose lift-curve to the slope of the configuration lift-curve,

$\bar{a}_w = \left(\frac{Cl_{\alpha_{wing}}}{Cl_{\alpha_{conf}}} \right)$, the proportion of the slope of the wing lift-curve to the slope of the configuration lift-curve, and

$\bar{a}_t = \left(\frac{Cl_{\alpha_{tail}}}{Cl_{\alpha_{conf}}} \right)$ the proportion of the slope of the tail-curve to the slope of the configuration lift-curve.

Similarly, we can normalise Eqn (6.2) using the static margin h, *as the normalising parameter for the moment arms* and the configuration lift curve slope $Cl_{\alpha_{conf}}$, *as the normalising parameter for the force terms*. Thus, we get the proportion of the aerodynamic moment contributed by each of the three lifting elements as a proportion of the total moment contributed by the configuration

$$(\bar{a}_n * \lambda_n) + (\bar{a}_w * \lambda_w) + (\bar{a}_t * \lambda_t) = 1 \tag{6.15}$$

where,

$\lambda_n = \frac{l_n}{h}$, the ratio of the moment arm for the nose lift to the static margin,

$\lambda_w = \frac{l_w}{h}$, the ratio of the moment arm for the wing lift to the static margin, and

$\lambda_t = \frac{l_t}{h}$, the ratio of the moment arm for the tail lift to the static margin.

Provided solutions exist, the solutions to these two normalised Eqns, (6.14) and (6.15), represent the complete family of normalised configuration solutions to the configuration design problem. Every normalised solution from this set can be *scaled up* subsequently, using specific values of the static margin h and the selected value of the configuration lift-curve slope $Cl_{\alpha_{conf}}$. Thus, the entire range of configuration solution possibilities to meet specific user requirement can thus be identified.

It is however noticed that there are six unknown configuration parameters — $\bar{a}_n, \bar{a}_w, \bar{a}_t, \lambda_n, \lambda_w, and \lambda_t$ and only two equations between them. Additional relations have to be found before these two equations can be solved uniquely. *The additional relations are provided by the constraint specifications.*

6.7 CONSTRAINT EQUATIONS

The requirement for static stability demands that the neutral point is located aft of the centre of gravity of the configuration. This implies that the *static margin, h should always be positive.*

6.7.1 Controllability Considerations

The controllability of the configuration is dependent upon the value of the gearing ratio. Its value can be positive or negative depending on the sign of the control arm l_C. Normalising Eqn (6.6), using the configuration lift-curve slope and the static margin as the normalising parameters, we obtain

$$\frac{\delta_c}{\alpha_{conf}} = -\frac{1}{\left(\lambda_c * \bar{a}_c\right)} \tag{6.16}$$

The angle of control deflection δ_c is usually designed to be greater than the angle of attack α_{conf}, to make the configuration less sensitive to the inherent noise in the control system. This requirement is assured by the condition:

$$\left|\left(\bar{a}_c * \lambda_c\right)\right| < 1 \tag{6.17}$$

In all efficient configuration designs $|\lambda_C|$ is usually selected greater than 1.0, so as to keep the control power requirement as low as possible. In addition, $|\bar{a}_C|$ should not only be positive but also less than 1.0. *Are there any bounds on the availability of the control force L_C ?*

6.7.2 Magnitude of Control Force, L_c

Till now the factors influencing the magnitude of the control arm have been discussed. The factors that influence the magnitude of the control force itself need to be considered. After distributing the configuration lift force on three lifting elements to satisfy the two system requirements, the designer chooses one of the lifting elements for purposes of controlling the configuration.

Is there any link between the control lift force produced by the chosen lifting element and the lift produced by it when, as part of the configuration, it is set at an angle of attack, α ?

A study of mechanics of the lift generation by the lifting element is relevant for understanding the link. It is well known from *Slender Body Theory*, that the lift force generated by an isolated lifting-surface(ls), set at an angle of attack α, will be augmented by two factors $K_{b(ls)}$ and $K_{ls(b)}$, when the same lifting surface, now forming part of a cylindrical configuration, is set at the same angle of attack, α Thus,

$$Cl_{\alpha_{(ls)}} = Cl_{\alpha}{}_{(\text{isolated ls})} * \left(K_{b(ls)} + K_{ls(b)}\right) \qquad (6.18)$$

These two factors are the Nielsen's[4] lift interference factors, which account for the presence of the body in augmenting the basic lift force of the isolated lifting surface. These two factors are functions of s/r, the ratio of semi span of the lifting surface to the radius of the body at the location of the lifting surface.

Similarly, the control lift force generated by the selected lifting surface is related to the basic lift force of the isolated lifting surface by relation shown in Eqn (6.19).

$$Cl_{\delta\,(ls)} = Cl_{\alpha\,(isolated\ ls)} * \left(k_{b(ls)} + k_{ls(b)}\right) \qquad (6.19)$$

These two factors $k_{b(ls)}$ and $k_{ls(b)}$ are also known as Nielsen's lift interference factors. These are also functions of the ratio s/r.

Using Eqns (6.18) and (6.19), the control lift generated on the selected lifting element can be related to the configuration lift distributed on it by the following relations:

$$\left.\begin{aligned}
\bar{a}_c &= \bar{a}_{ca} * \frac{k_{b(ca)} + k_{ca(b)}}{K_{b(ca)} + K_{ca(b)}} \text{, in the case of canard control}\\
\bar{a}_c &= \bar{a}_w * \frac{k_{b(w)} + k_{w(b)}}{K_{b(w)} + K_{w(b)}} \text{, in the case of wing control}\\
\bar{a}_c &= \bar{a}_t * \frac{k_{b(t)} + k_{t(b)}}{K_{b(t)} + K_{t(b)}} \text{, in the case of tail control}
\end{aligned}\right\} \qquad (6.20)$$

Equation (6.20) leads to the determination of the ratios,

$\frac{\bar{a}_c}{\bar{a}_{ca}}, \frac{\bar{a}_c}{\bar{a}_w} and \frac{\bar{a}_c}{\bar{a}_t}$ as functions of the s/r ratio of the lifting elements. In efficient weapon designs, every effort is taken to minimise the control force requirement, since this has a direct bearing on the availability of control power. The control force can be minimised only by minimising the lift interference parameters within the parenthesis on the right hand side of Eqn (6.20). The factors within the parenthesis are functions of the ratio s/r. This functional dependency is plotted in Fig. 6-3 and also tabulated in Table 6-3. It

Table 6-3. Lift interference factors as a function of (s/r) ratio

r/s	s/r	$K_{ls(b)}$	$K_{b(ls)}$	$\left(\frac{K_{b(ls)}+K_{ls(b)}}{k_{b(ls)}+k_{ls(b)}}\right)$	$\left(\frac{k_{b(ls)}+k_{ls(b)}}{K_{b(ls)}+K_{ls(b)}}\right)$
1.0	1.0	2.0	2.0	2.0	0.5
0.80	1.25	1.773	1.467	1.827	0.547
0.666	1.5	1.627	1.151	1.708	0.586
0.50	2.0	1.450	0.799	1.551	0.645
0.40	2.5	1.349	0.611	1.453	0.688
0.333	3.0	1.284	0.493	1.384	0.722
0.286	3.5	1.239	0.414	1.333	0.750
0.25	4.0	1.206	0.356	1.295	0.772
0.20	5.0	1.162	0.278	1.240	0.807
0.125	8.0	1.097	0.168	1.153	0.867
0.10	10.0	1.077	0.133	1.124	0.890
0.0	∞	1.0	0.0	1.0	1.0

$$K_{ls(b)} = \frac{\frac{2}{\pi}\left(\left(1+\left(\frac{r}{s}\right)^4\right)\left(\frac{1}{2}\tan^{-1}\frac{1}{2}\left(\frac{s}{r}-\frac{r}{s}\right)+\frac{\pi}{4}\right)-\left(\frac{r}{s}\right)^2\left(\left(\frac{s}{r}-\frac{r}{s}\right)+2\tan^{-1}\frac{r}{s}\right)\right)}{\left(1-\frac{r}{s}\right)^2}$$

$$K_{b(ls)} = \frac{\left(1-\left(\frac{r}{s}\right)^2\right)^2 \frac{2}{\pi}\left(\left(1+\left(\frac{r}{s}\right)^4\right)\left(\frac{1}{2}\tan^{-1}\frac{1}{2}\left(\frac{s}{r}-\frac{r}{s}\right)+\frac{\pi}{4}\right)-\left(\frac{r}{s}\right)^2\left(\left(\frac{s}{r}-\frac{r}{s}\right)+2\tan^{-1}\frac{r}{s}\right)\right)}{\left(1-\frac{r}{s}\right)^2}$$

$$K_{ls(b)} = k_{ls(b)} + k_{b(ls)}$$

is seen that the factor $\left(\frac{k_{b(ls)}+k_{ls(b)}}{K_{b(ls)}+K_{ls(b)}}\right)$ varies from 0.5 to its asymptotic value of 1.0, as the s/r ratio, varies from 1 to ∞. The maximum reduction occurs in the range of s/r from 1.0 to 3.5. The ratio of 1.0, however, is the theoretical lower limit when the body acts as a reflection plane, and there is no physical control surface at all. This limiting ratio has no real significance in design. Usually, in all weapon configuration designs the range in the ratio of s/r for the control element is

$$2.0, \le \frac{s}{r} \le 3.5 \tag{6.21}$$

In this range of s/r ratio, the range in the value of $\left(\frac{k_{b(ls)}+k_{ls(b)}}{K_{b(ls)}+K_{ls(b)}}\right)$ is given by

$$0.645 \le \left(\frac{k_{b(ls)}+k_{ls(b)}}{K_{b(ls)}+K_{ls(b)}}\right) \le 0.75 \tag{6.22}$$

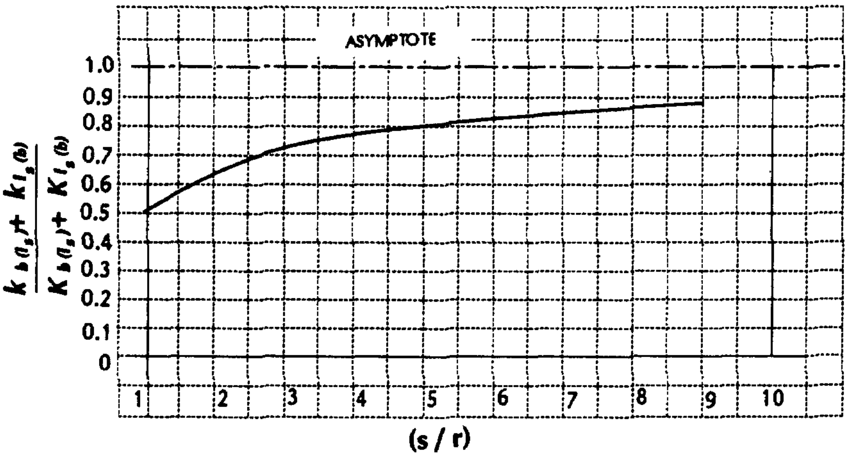

Figure 6-3. $\left(\frac{k_{b(ls)}+k_{ls(b)}}{K_{b(ls)}+K_{ls(b)}}\right)$ **as a function of (s/r)**

6.7.3 Generation of Manoeuvring Force

The manoeuvre force generated on the configuration is given by the Eqn (6.7). It is noticed that the configuration lift force is modified by the factor $\left(1 - \frac{h}{l_c}\right)$ to generate the requisite manoeuvring force. In the case of tail-controlled configurations, for positive values of the gearing ratio, the configuration lift force is diminished by this factor. Whereas for negative values of the gearing ratio, as in the case of canard-controlled and wing-controlled configurations, the configuration lift force is increased by this ratio. In efficient tail-controlled weapons, however, this inevitable diminution needs to be minimised by proper selection of the gearing ratio.

In Fig. 6-4, the variations of the control effectiveness factor $(1-h/l_c)$ have been presented as a function of the gearing ratio. It is seen that in tail-controled weapons the effectiveness *increases asymptotically* to its value of 1, for values of l_c greater than 4. Even when the gearing ratio is doubled from 4 to 8, the controlled effectiveness is found to increase only gradually from 0.75 to 0.875. Therefore, in tail-control configurations, the usual practice is to limit the gearing ratio to around 8. In canard-controlled configurations, however, the effectiveness *decreases asymptotically* to 1.0, as $|l_c|$ is increased to values greater than 4. Here also we notice that when $|l_c|$ is doubled from 4 to 8, the effectiveness decreases only gradually from 1.25 to 1.125. It reaches its asymptotic value of 1, as the gearing ratio tends to infinity. Therefore, just as in tail-control configurations, in the case of canard-control configurations also, the practice is to limit the canard control arm to 8.

Wing-controlled configurations are treated as a special variation of the canard-controlled configurations where, the control surfaces are placed very close to the centre of gravity of the configuration. This favourable location increases the control effectiveness asymptotically to infinity, when the magnitude of the control arm $|l_c|$ decreases to zero.

Based on the above considerations, the constraint on the selection of λ_c, can be specified as follows:

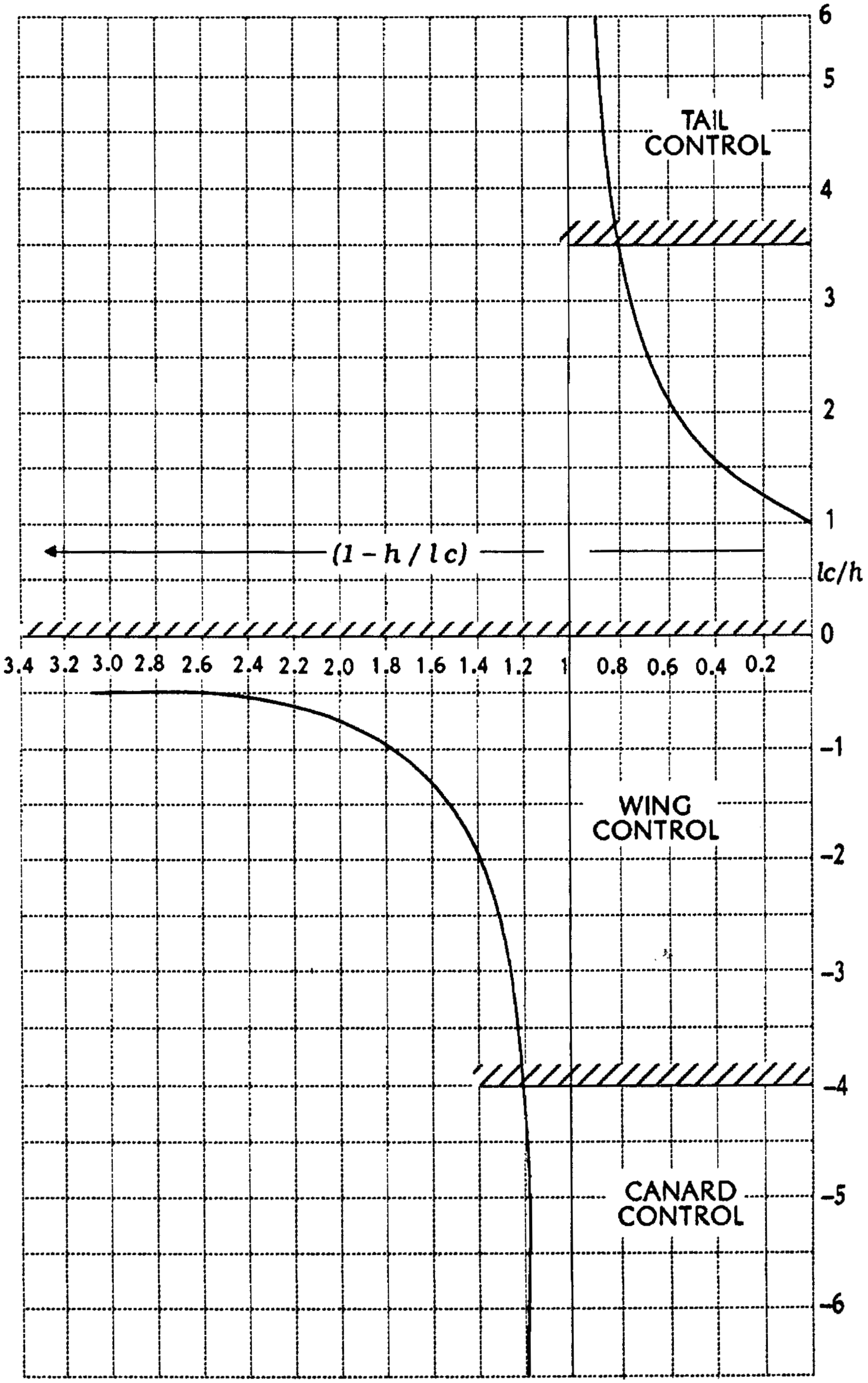

Figure 6-4. Control effectiveness as a function of (l_c / h)

In tail-controlled and canard-control configurations,

$$4.0 < |\lambda_c| < 8.0. \tag{6.23}$$

In wing control configurations,

$$-1.0 < \lambda_c < 0 \tag{6.24}$$

6.7.4 Limits of Stalling of Canard-Control Surfaces

In canard-controlled configurations, the sense of the canard deflection is the same as the sense of the angle of attack of the configuration. Therefore, particularly when the angle of attack demanded on the configuration is high, the canards would present a higher angle of attack to the wind stream. This could take the canards to their limits of stalling. While selecting the gearing ratio therefore, care should be taken to avoid stalling of the control surfaces. Equation (6.16) gives the relation between the angle of control deflection and the trim angle of attack of the configuration, as a function of the gearing ratio. In canard-controlled configuration, the angle presented to the wind stream by the control surfaces is the sum of the angle of attack of the configuration and the angle of deflection of the control surfaces. This angle of the canard control surfaces should satisfy the inequality:

$$\left(\alpha_{conf} + \delta_c\right) \leq \textit{stalling limit}$$

This requirement leads to the constraint equation on the selection of the control moment, $(\bar{a}_c * \lambda_c)$ as follows:

$$\left(\frac{\textit{Stall limit}}{\alpha_{conf}}\right) \geq \left(1 + \frac{1}{(\bar{a}_c * \lambda_c)}\right) \tag{6.25}$$

6.7.5 Constraint on Nose Lift

It is well known from slender body theory that any tapering cylindrical forebody will generate lift, the slope of its lift curve being

$$Cl_{\alpha_{nose}} = 2.0 \,/\, radian \,\cdot$$

However, in ogive-cylinder and cone-cylinder bodies the slope of the lift curve, in general, is greator than 3.0/radian. In almost all practical configurations, the configuration lift curve slope, $Cl_{\alpha_{conf}}$, is usually greater than 10/radian. Therefore, the expected range in the value of $\bar{a}_n$ is obtained as:

$0.0 < \bar{a}_n \leq 0.3.$

In the case of blunt-nosed configurations however, the forebody would not contribute any lift.

Based on the above considerations, depending on whether the configuration has a lifting forebody or not, we can define the constraint in the ratio of the nose lift to the configuration lift by the relations (6.26a) or (6.26b)

In the case of tapered lifting forebodies,

$$0.0 < \bar{a}_n \leq 0.3. \tag{6.26a}$$

In the case of blunt non-lifting forebodies,

$$\bar{a}_n = 0.0 \tag{6.26b}$$

6.7.6. Limits on the Moment Arm for the Nose Lift

Purely from controllability considerations, one cannot specify the limits on the moment arm for the nose lift. However, it is well known that the longer the moment arm, greater is the bending moment caused by the nose lift. Therefore the constraint on the moment arm for the nose lift should come from structural considerations. For the present, the values usually employed in most of the current tactical guided weapon are assumed.

$$-8.0 < \lambda_n < -3.0 \tag{6.27}$$

6.8 NORMALISED CONFIGURATION SOLUTIONS

It is noticed now that the governing equations and the constraint equations are all functions of normalised variables. It is therefore possible to identify all configuration solutions to meet the user's stated need, in the normalised plane itself. This normalised solution set is the *universal set* of solutions to the configuration design problem. Depending upon the choice of one of the three lifting elements for control purposes, one gets solutions to canard-controlled configurations, wing-controlled configurations or tail-controlled configurations. The dimensioned level configuration solutions can then be obtained by scaling up the normalised solutions, using the static margin h, and the selected value of the configuration lift-curve slope $Cl_{\alpha_{conf}}$, that are specific to the particular design. For this reason, both h and $Cl_{\alpha_{conf}}$ are called the *scaling parameters*.

Use of the above technique to identify all tail-controlled and canard-controlled configurations to meet a user's stated need has been included in *Appendix D.* Having identified all the solutions to meet the user's needs, a question naturally arises as to which type of configuration one should choose from the different available options. This question is discussed below.

6.9 TAIL-CONTROLLED VS CANARD-CONTROLLED CONFIGURATIONS

Both tail-controlled and canard-controlled configuration solutions have been found to meet the systems level demands on the configuration. *Is there any special merit in going in for one type of configuration, in preference to the other?*

To answer this question, one needs to take a closer look at Eqn (6.7) relating the manoeuvre force with the configuration lift force. As has been detailed earlier, the factor $(1-h/lc)$ modifies the configuration lift force before it is transformed into the manoeuvring force. For positive values of the gearing ratio (h/lc) as in tail-controlled configurations, the achievable manoeuvre force is always less than the configuration lift force. On the other hand, for negative values of the gearing ratio, the manoeuvre force obtainable is always greater than the configuration lift force. This advantage makes it possible for canard-control configurations, to achieve the required manoeuvre force at a lesser angle of attack than a tail-control configuration. This is illustrated in Fig. 6.5 which is a plot of Eqn (6.7). This clearly shows the advantage of a canard-control configuration over a tail- control configuration. *What can be achieved by a tail-control configuration, can be achieved, at a lower angle of attack by a canard configuration, but the vice-versa is not always true.*

However, as indicated in Chapter 1, every configuration solution should be evaluated before a final choice is made by the user. All solutions being equal in meeting the user's performance expectations, the final choice is based on the user's acceptability criterion. This consideration is not based on technical reasons alone, but is based on other considerations such as the economics of operation, ease of maintainability, environmental and user-friendly features, *etc.*

6.10 CONCLUSION

We were amazed at the variety of configuration shapes listed in Janes Weapon Systems for guided weapons systems. We set out to find the logic behind the choice of such a variety of configuration shapes. We have now succeeded in finding the answers to these questions.

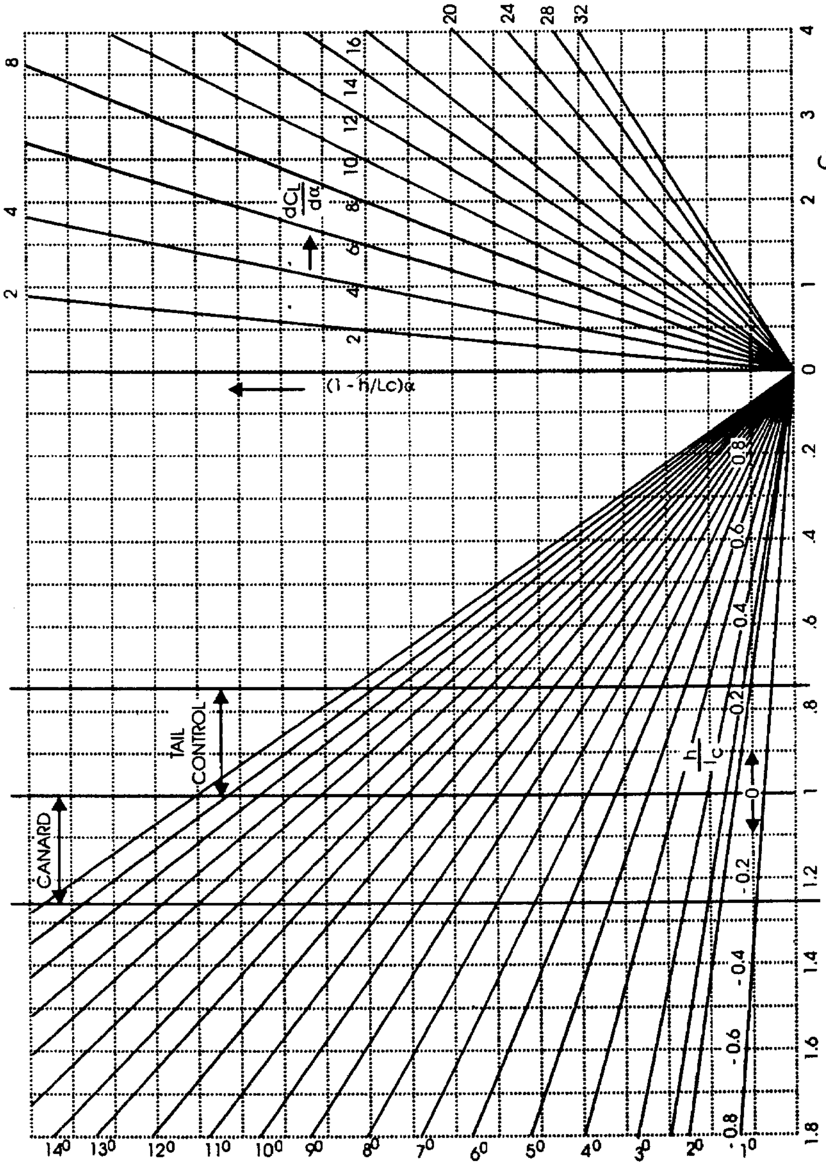

Figure 6-5. Advantage of a canard over tail-control

The key lay in the application of the systems study methodology to weapon systems design. We noticed that a hierarchy existed in guided weapons design. The first level in the hierarchy, called the outer loop, reflected the perception of the systems level designer. He viewed the complete weapon system as the interaction between two point masses, one moving under the command of the adversary and the other moving under the command of the chosen guidance policy. The application of the systems study methodology at this stage enabled the systems designer to identify two systems level requirements for the design of the aerodynamic configuration. There are:

- The amount of lateral force demanded on the configuration to meet its manoeurving needs.
- The amount of lag that the system would tolerate.

At this stage the configuration designer took on the task of identifying all the configuration solutions that met the above two systems requirements. For this purpose, the designer modelled the weapon configuration, essentially as a long cylinder with three lift producing elements located at three distinct locations from its centre-of-gravity. He also took recourse to the same systems study methodology for the identification of all the configuration possibilities, but at this stage the methodology was applied recursively at the sub-system level.

It was found to be advantageous at this stage, to normalise the governing sub-system equations and its associated constraint equations to find all the normalised configuration solutions that met the above two systems level requirements. These normalised solutions are the universal set of solutions to the configuration design problem. After identifying this universal set, each of these solutions could be scaled up, using the scaling parameters relevant to specific designs. Thus all the practical weapon system configuration solutions could be identified.

The final choice out of these configuration possibilities was, however, made by the user depending upon his acceptance criterion.

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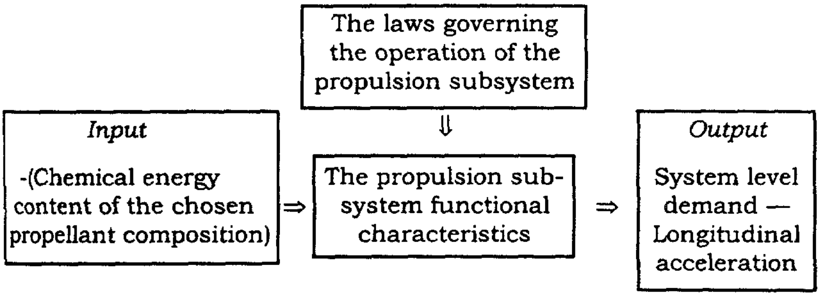
CHAPTER 7

PROPULSION SYSTEM DESIGN

The magnitudes of the longitudinal acceleration necessary for assuring the required velocity profile for the weapon system were determined by conducting the systems study on the outer loop. This input is the starting point for the design of the propulsion subsystem.

In this chapter, a method is developed for the determination of the essential propulsion subsystem characteristics that link the design of the propulsion subsystem with the longitudinal acceleration demanded on it by the outer loop. System study procedure is adopted for selecting the compatible set of these subsystem parameters. Following Dixon[1] the four specific design elements needed for conducting such systems study are tabulated in Table 7-1.

Table 7-1. Design elements for propulsion system



As a first task, therefore, these subsystem characteristics need to be identified. Systems study methodology comes to our rescue once again for identifying the essential subsystem characteristic needed to meet the the overall system level demands. Propulsion system design is then concerned with the implementation of these subsystem characteristics in the design, within the constraints under which the designer seeks a solution.

As is usual in all systems studies, the designer's perception of the subsystem is the first aspect to be studied.

7.1 DESIGNER'S PERCEPTION OF PROPULSION SYSTEM

The mass of the warhead required to achieve a specified value of single shot kill probability has already been assessed by the warhead subsystem designer, depending upon the miss distance achievable by the chosen guidance policy. The propulsion systems designer therefore, views his subsystem as the essential element in the weapon design to convey the warhead to the target at the speed demanded by the guidance policy. To impart the needed speed to the weapon, the propulsion system should provide the longitudinal acceleration, demanded by the chosen guidance policy. Considering this perception, the problem for the propulsion systems designer can be formulated as follows:

Device a minimum weight solution for the propulsion system for imparting the needed longitudinal acceleration to the weapon, within the constraint of the available technology for its design.

Having, defined his objectives, the propulsion sub-system designer starts his task with the following perception of his sub-system.

The designer perceives the propulsion system essentially as a device that converts the chemical energy of the propellants into mechanical energy, thus providing the necessary thrust force. The magnitude of the thrust force should be adequate to accelerate the weapon against the retarding aerodynamic drag forces. Applying Newton's Law, the relation that expresses this perception of the designer is expressed by the following relation.

$$\text{Mass} * \text{Longitudinal acceleration} = \text{Thrust} - \text{Drag} \tag{7.1}$$

At this stage of design, the systems designer has no knowledge of the thrust force requirement, since the mass of the weapon is not known. Similarly, he does not know the drag force that decelerates the weapon since the configuration shape is not known. The systems designer, therefore, recasts the above relation in terms of the longitudinal acceleration, since this is the parameter that the weapon system demands from the propulsion subsystem. The recast relation is given by Eqn (7.2).

$$\text{Longitudinal acceleration} = \left(\frac{\text{Thrust} - \text{Drag}}{\text{mass}}\right) \tag{7.2}$$

The conversion of the chemical energy of the propellant into useful thrust force is given by the rocket equation

$$\text{Thrust} = I_{sp} * \frac{m_p}{t_b} * g \tag{7.3}$$

The drag force is expressed by the aerodynamic relation

$$Drag = \frac{1}{2} * \rho * V^2 * S * C_d \tag{7.4}$$

Using Eqns (7.3) and (7.4) in Eqn (7.2), the demanded longitudinal acceleration can be expressed as:

$$\text{Longitudinal acceleration} = \left(\frac{\left(\frac{I_{sp}}{t_b}\right) * g * \left(\frac{m_p}{m_0}\right)}{1 - \left(\frac{m_p}{m_0}\right) * \left(\frac{t}{t_b}\right)} \right) \left(\frac{\left(\frac{V^2 * C_d}{\mu_0}\right)}{1 - \left(\frac{m_p}{m_0}\right) * \left(\frac{t}{t_b}\right)} \right) \tag{7.5}$$

Eqn (7.5) is the required relation linking the demanded lateral acceleration with the characteristic functional features of both the propulsion system and the decelerating characteristics of the configuration. The parameter $\left(\frac{C_d * V^2}{\mu_0}\right)$ defines the drag deceleration characteristics of the configuration and the non-dimensional ratios $\left(\frac{I_{sp}}{t_b}\right)$ *and* $\left(\frac{m_p}{m_o}\right)$ define the functional features of the propulsion system. The propulsion designer will have to select the appropriate range in the values of these propulsion functional features to find a minimum weight solution for the propulsion system. Systems study methodology is used to at this stage to determine the ranges in the values of these parameters. The only parameter influencing the longitudinal acceleration is the drag deceleration characteristics of the configuration. Since this aspect is under the purview of the aerodynamics designer, the propulsion designer will have to work closely with his counterpart from the aerodynamic configuration design team at this stage.

7.2 PROPULSION POLICY

It has been discussed in the earlier chapter that the longitudinal acceleration, demanded by the chosen guidance policy on the system, can be achieved by one of two propulsion thrust policies. These are:

Case 1. Boost-sustained thrust policy, and

Case 2. Boost-coast thrust policy.

The first task of the propulsion systems designer is to relate the propulsion subsystem characteristics, specific to the chosen thrusting policy, with the system level demands of the longitudinal acceleration on the weapon.

7.2.1 Case 1. Boost Sustained Policy

In this policy, during the boosting phase of flight, the weapon is accelerated to its operational speed in the quickest possible time. Thereafter, the speed of the weapon is maintained essentially constant during its sustained phase of flight. The specific needs of the propulsion sub-system in these two phases of flight are discussed in the reverse order; the requirements of the sustained phase of flight first and then the requirements of the boosting phase.

7.2.1.1 Sustained phase of flight

In the sustained phase of flight, the propulsion system is needed to maintain a constant speed for the weapon till its interception with the target, overcoming the drag forces. The thrust of the propulsion system, in this phase of flight, needs to be *just sufficient* to overcome the aerodynamic drag associated with the flight speed of the weapon. No longitudinal acceleration is therefore, needed in this phase of flight. Reflecting this need, Eqn (7.5) is therefore modified as:

$$\left(\frac{\left(\frac{I_{sp_s}}{t_{b_s}} \right) * g * \left(\frac{m_{p_s}}{m_{be}} \right)}{1 - \left(\frac{m_{p_s}}{m_{be}} \right) * \left(\frac{t}{t_{b_s}} \right)} \right) = \left(\frac{\left(\frac{V_s^2 * C_d}{\mu_{be}} \right)}{1 - \left(\frac{m_{p_s}}{m_{be}} \right) * \left(\frac{t}{t_{b_s}} \right)} \right) \tag{7.6}$$

where,

I_{sp_s} is the specific impulse of the propellant used in the sustainer phase

m_{p_s} is the mass of the propellant burnt in the sustainer phase

m_{be} is the boost-end mass of the weapon

t_{b_s} is the time of burning of the propellant in the sustained phase of flight of the weapon

V_s is the speed of the weapon in the sustainer phase of flight. This is the same as V_{be}, the speed of the weapon at the end of the previous boosting phase of flight.

C_d is the drag coefficient of the weapon configuration at the sustained speed, V_s

$$\mu_{be} = \frac{2 * m_{be}}{\rho * S}$$

The propellant mass ratio needed in the sustained phase of flight, (in order to maintain a constant speed overcoming the attendant aerodynamic drag force), is got from Eqn (7.6) as:

$$\left(\frac{m_{ps}}{m_{be}}\right) = \left(\frac{C_d}{I_{sp_s} * \mu_{be}}\right) * \left(\frac{V_s^2 * t_s}{g}\right) \tag{7.7}$$

The magnitude of sustained speed, V_s and the duration of sustained flight, t_s have been determined by the systems study of the outer loop. Therefore, for specified values of V_s *and* t_s the dependence of the propellant mass ratio $\left(\frac{m_{ps}}{m_{be}}\right)$, on $\left(\frac{C_d}{I_{sp_s} * \mu_{be}}\right)$ is a rectangular hyperbola. This functional dependence is shown in Fig. 7-1, as a graphical nomogram.

Two rectangular hyperbolae have been drawn for $\left(\frac{V_s^2 * t_s}{g}\right)$ of 45,000 and 250,000 respectively. These values cover the typical range applicable to short range tactical weapon systems. For specific value of $\left(\frac{V_s^2 * t_s}{g}\right)$, the propulsion mass ratio needed in the sustained phase of flight can be obtained as a function of C_d, *within the constraint of achievable* I_{sp_s}.

It is seen from Fig. 7-1, that most of the range in the values of $\left(\frac{V_s^2 * t_s}{g}\right)$ is covered by a propellant mass ratio $\left(\frac{m_{ps}}{m_{be}}\right) = 0.05$. The range in the value of μ_{be} can then be read off the nomogram, corresponding to the value of C_d achievable on the configuration. It is seen from the nomogram that for a weapon configuration having a drag coefficient *even as high as* $C_d = 0.5$, the value of μ_{be} would

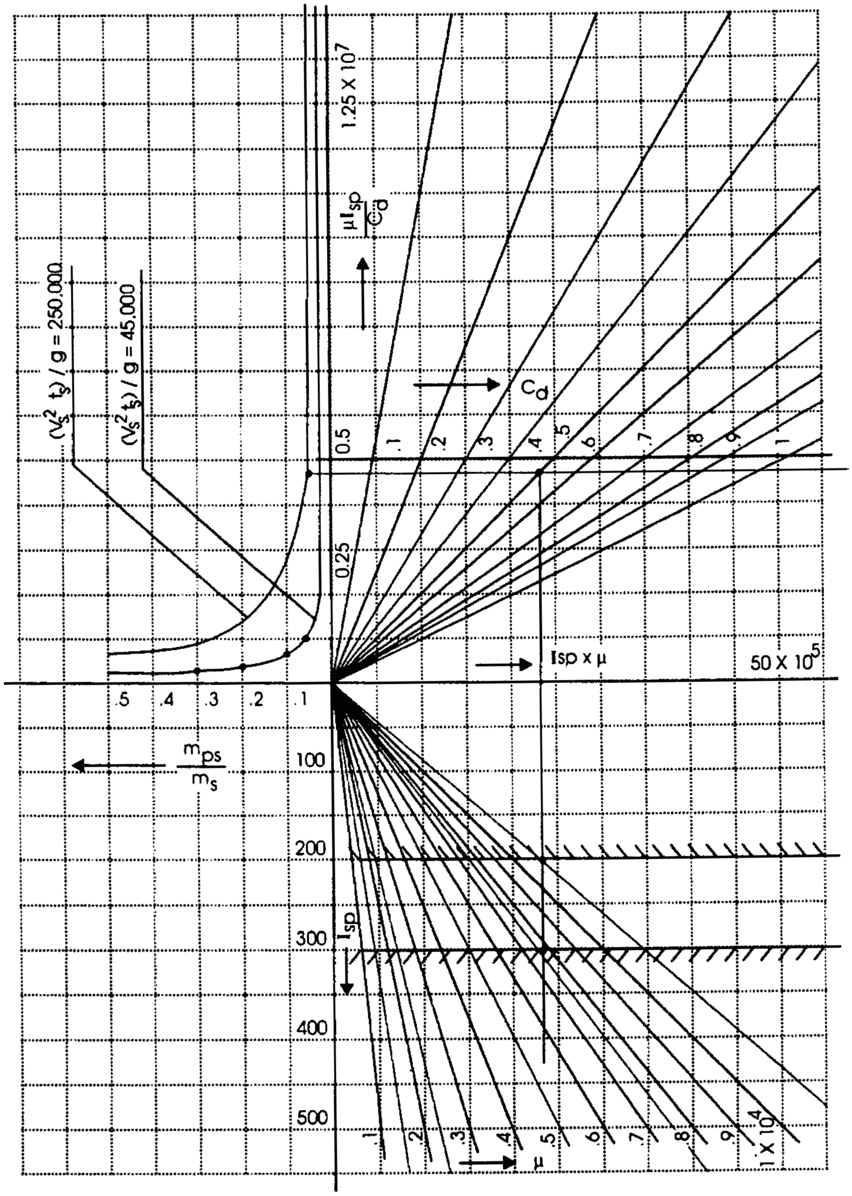


Figure 7-1. Nomogram for propulsion system design

range from $\mu_{be} = 0.15 \times 10^4$ for $\left(\frac{V_s^2 * t_s}{g}\right) = 45000$, to $\mu_{be} = 1.25 \times 10^4$ for $\left(\frac{V_s^2 * t_s}{g}\right) = 250000$. These values cover I_{sp_s} ranging from 200 to 300 s.

7.2.1.2 Boosting phase

In this phase of flight, the propulsion device needs to accelerate the weapon to its boost-end speed in the quickest possible time, within the constraint of available propellant technology. Drag deceleration during this phase *can be neglected* compared to the thrust acceleration. Equation (7.5), relevant for this phase of flight would therefore be modified as follows:

$$\text{Longitudinal acceleration} = \left(\frac{\left(\frac{I_{sp_b}}{t_{bb}}\right) * g * \left(\frac{m_{p_b}}{m_0}\right)}{1 - \left(\frac{m_{p_b}}{m_0}\right) * \left(\frac{t}{t_{bb}}\right)}\right) = \frac{dV}{dt} \tag{7.8}$$

This is the famous rocket equation. The variables in this equation are defined below.

I_{sp_b} is the specific impulse of the propellant used in the boosting phase

m_{p_b} is the mass of the propellant burnt in the boosting phase

m_0 is the mass of the weapon at the start of the boosting period - it is the all-up mass of the weapon

t_{bb} is the time of burning of the propellant in the boosting phase of flight of the weapon

g is the acceleration due to gravity

The above equation can be integrated to get the propellant mass ratio in the boosting phase to meet the requirement of increment in the speed of flight as a function of specific impulse. The propellant mass ratio is obtained as

$$\left(\frac{m_{p_b}}{m_0}\right) = e^{-\left(\frac{V_s - V_i}{I_{sp_b} * g}\right)} \tag{7.9}$$

Usually, V_i is zero, when the weapon is launched from a static launcher. Therefore,

$$\left(\frac{m_{p_b}}{m_0}\right) = e^{-\left(\frac{V_s}{I_{sp_b} * g}\right)} \tag{7.10}$$

7.2.1.3 Assessment of the mass of weapon

The final mass that is delivered on the target, at the end of the sustained phase of flight, consists of the mass of the warhead and the other non-propulsive masses. This term would include the masses of the structural elements, the mass of the guidance and control units and the mass of the power supplies. Thus,

$$m_{final} = \left(m_{be} - m_{ps}\right) = \left(m_{warhead} + m_{non-propulsive}\right) \tag{7.11}$$

If we define the non-propulsive mass of the weapon as a ratio of the mass of the warhead, then we can define the figure of merit of the system as:

$$\eta = \frac{m_{non-propulsive}}{m_{warhead}}, \tag{7.12}$$

Using (7.9) we can recast Eqn (7.8) in terms of η as Eqn

$$m_{be} * \left(1 - \frac{m_{ps}}{m_{be}}\right) = m_{warhead} * (1 + \eta) \tag{7.13}$$

The ratio of the boost-end mass of the weapon to that of the warhead can be defined as a function of the figure of merit. Thus,

$$\frac{m_{be}}{m_{warhead}} = \left(\frac{1 + \eta}{1 - \frac{m_{ps}}{m_{be}}}\right) \tag{7.14}$$

The propellant requirement in the previous phase of flight, which is the boosting phase of flight is discussed below.

The mass of the weapon at the end of boosting is m_{be}. This is now expressed by the relation

$$m_{be} = \left(m_0 - m_{p_b}\right) = m_0 * \left(1 - e^{-\frac{V_s - V_i}{I_{sp_b} * g}}\right) \tag{7.15}$$

Expressing the total mass of the weapon as a ratio of the mass of the warhead, we get

$$\left(\frac{m_0}{m_{warhead}}\right) = \left(\frac{m_0}{m_{be}}\right) * \left(\frac{m_{be}}{m_{warhead}}\right) = \left(\frac{1}{1 - e^{-\frac{V_s - V_i}{I_{sp_b} * g}}}\right) * \left(\frac{1 + \eta}{1 - \frac{m_{p_S}}{m_{be}}}\right) \tag{7.16}$$

Equation (7.16) is the relation that enables one to make a preliminary assessment of the total mass of the proposed weapon.

7.2.2 Case 2. Boost Coast Policy

This policy demands that the weapon is accelerated to a very high speed from its initial speed in the boosting phase. Thereafter, in its coasting phase of flight, the boost-end momentum of the weapon carries it to its final destination overcoming the aerodynamic deceleration. As in the consideration of the boost-sustained policy, in this case also, the coasting phase is considered first and then the boosting phase.

7.2.2.1 Coasting phase of flight

In this phase of flight, the boost-end momentum of the weapon is expended in overcoming the aerodynamic drag of the weapon configuration. Equation (7.5) relevant for this phase of flight is modified as:

$$Longitudinal\ acceleration = -\frac{V^2 * C_d}{\mu_{be}} \tag{7.17}$$

It is seen that the only way to minimise the drag deceleration term is to minimise the right hand side of Eqn (7.17). This would imply either minimising the numerator or maximising the denominator. The drag coefficient of the configuration is a function of its flight Mach number. In the supersonic regions of flight, as the speed decelerates, the drag coefficient increases to its maximum value in the transonic region. Since at the systems study stage, the details of the weapon configuration are not available, it is worthwhile analysing the effect of increasing the denominator. For a specified configuration, increasing the value of μ_{be} beyond 6000 does not significantly vary the drag deceleration term. Therefore in coasting configurations μ_{be} *should be kept greater than 6000* and every effort is made during the detailed design of the configuration to minimise the numerator term. Efforts should also be made to avoid the weapon decelerating to the transonic range in the coasting period.

7.2.2.2 Boosting phase

The analysis of this phase of flight is identical to the one developed while discussing the boost-sustained policy. Therefore it is not repeated here. From Eqn (7.15), the ratio of the boost-end mass to the all-up mass is obtained as:

$$\frac{m_{be}}{m_0} = 1 - e^{-\left(\frac{V_{be} - V_i}{I_{sp_b} * g}\right)} \tag{7.18}$$

The mass at boost-end comprises the mass of the warhead and the mass of the non-propulsive units. Of these two masses, the mass of the warhead is a requirement on the system that has been arrived by conducting systems study of the overall system. Therefore, the boost-end mass can be expressed as a function of the mass of the warhead and the figure of merit.

Using Eqn (7.11) in Eqn (7.18), the all-up mass of the weapon can be expressed in terms of the mass of the warhead and the figure of merit as follows:

$$\frac{m_0}{m_{warhead}} = \left(\frac{1 + \eta}{1 - e^{-\left(\frac{V_{be} - V_i}{I_{sp_b} * g}\right)}}\right) \tag{7.19}$$

Equation (7.19) is the relation that enables one to make a preliminary assessment of the total mass of the proposed weapon.

7.3 CONCLUSION

In the case of the boost-sustained propulsion policy, Eqn (7.6) is the relation that links the systems requirement of V_s with the propulsion subsystem functional features such as I_{sp}, $\frac{m_p}{m_o}$, and μ Equation (7.16) is the relation that enables one to make a preliminary assessment of the total mass of the proposed weapon. It is noticed that the total mass is expressed as a ratio of the mass of warhead, *which is an input from the systems study of the outer loop.* The technological constraints faced by the designer are expressed by the figure of merit η, and the value of the specific impulse I_{sp}. For every assessed mass of the overall weapon, its physical size can be determined from the acceptable value of the factor μ. Thus, the complete set of essential propulsion subsystem functional requirements have been determined.

Similarly, in the case of a weapon following the boost-coast propulsion policy, Eqn (7.19) enables one to arrive at the all-up mass of the weapon. The propulsion subsystem functional features can then be arrived at by using Eqn (7.18). Thus the method developed in this chapter enables the system designer of the guided weapon to determine the essential functional requirement of the propulsion subsystem. The functional features that are essential for the detailed design of the propulsion subsystem have been identified. These are as follows:

In the case of boost-sustain propulsion policy,

$$\left(\frac{m_{p_s}}{m_{be}}\right), \left(\frac{m_{be}}{m_{warhead}}\right), \left(\frac{m_{p_b}}{m_0}\right), \left(\frac{m_0}{m_{warhead}}\right), \mu$$

In the case of a boost-coast propulsion policy

$$\left(\frac{m_{be}}{m_0}\right), \left(\frac{m_0}{m_{warhead}}\right), \mu$$

REFERENCE

1. Dixon, R. John (1980). Design engineering—inventiveness, analysis and decision making. Tata McGraw Hill Comany Ltd., New Delhi.

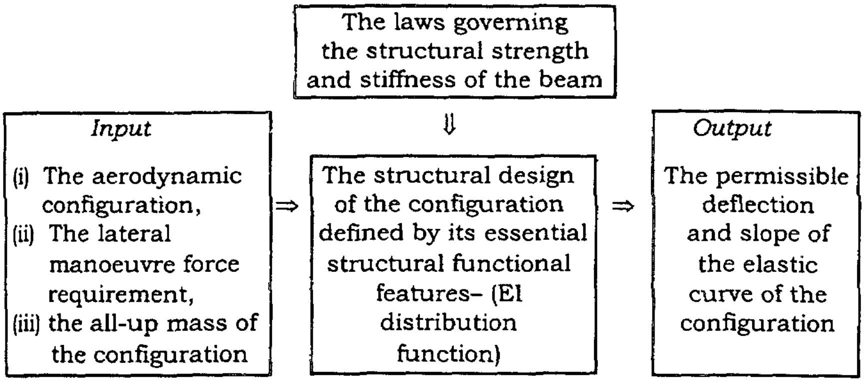
CHAPTER 8

STRUCTURAL DESIGN OF CONFIGURATION

The structural design of the configuration deals with the design of the external aerodynamic shape of the configuration to withstand the loads it is subjected to, as it flies towards the target. Structural design ensures the needed stiffness and the structural strength requirement on the structure of the aerodynamic configuration, within the constraints of permissible structural weight, availability of the material and the processes for its fabrication.

Like in all design problems, the first task of the structural systems designer is the determination of the essential structural functional features of the configuration by conducting systems study. Following Dixon[1] the four specific design elements needed for conducting such systems study are depicted in Table 8-1.

Table 8-1. Design elements of configuration



The laws governing the structural strength and stiffness of the beam ⇓

Input

(i) The aerodynamic configuration,
(ii) The lateral manoeuvre force requirement,
(iii) the all-up mass of the configuration

⇒ The structural design of the configuration defined by its essential structural functional features– (EI distribution function) ⇒

Output

The permissible deflection and slope of the elastic curve of the configuration

As usual, this activity starts with understanding the perception of the structural designer towards this problem.

8.1 STRUCTURAL DESIGNER'S POINT OF VIEW

The structural designer, like the aerodynamic configuration designer, views the configuration essentially as a long cylinder that is non-uniform both in its mass and stiffness distribution. On this cylinder, the aerodynamic configuration designer has already distributed the aerodynamic lifting force on three lifting elements, and chosen their locations on the body to satisfy the needs for the required static margin. The structural designer therefore, perceives this cylinder as a *beam in equilibrium under the influence of the three distributed aerodynamic lift forces, and the inertia load acting through its centre of gravity.* This perception of the structural designer is illustrated in Fig. 8-1. The beam would bend under the influence of these loads. The elastic deflection curve of the beam is also shown in the figure deflected under the influence of this loading pattern. The deflection curve is drawn as a function of the structural stiffness parameter (*E*I*) The mathematical relations governing the shape of the elastic deflection curve of the beam under the bending forces applied on the configuration are listed below. Using these equations, the required stiffness distribution along the configuration can be determined which satisfies the deflection and slope requirements that can be tolerated at specific locations on the configuration.

The bending moment along the configuration as a function of *EI*, is given by

$$EI * \frac{d^2y}{dx^2} = L_n * (x - x_n) + L_w * (x - x_w) + L_t * (x - x_t) - nW * (x - x_{cg}) \quad (8.1)$$

The slope of the elastic curve is given by

$$\frac{dy}{dx} = \frac{Area\ of\ the\ bending\ moment\ curve}{EI} \quad (8.2)$$

The deflection of the elastic curve is given by

$$y = \frac{Moment\ of\ the\ area\ of\ the\ the\ bending\ moment\ curve}{EI} \quad (8.3)$$

We can divide the configuration into a number of piecewise continuous segments, each of which has a different *EI* value. The selection of the *EI* value on each of the segments can then be tailored to satisfy the constraint on the slope and deflection over these segments.

At this stage it is necessary to consider, how the slope and deflection of the elastic curve constrain the proper functioning of some of the subsystems that are located inside the configuration. The functioning of two of the subsystems is taken up to illustrate the significance of the nature of the elastic curve on their functioning.

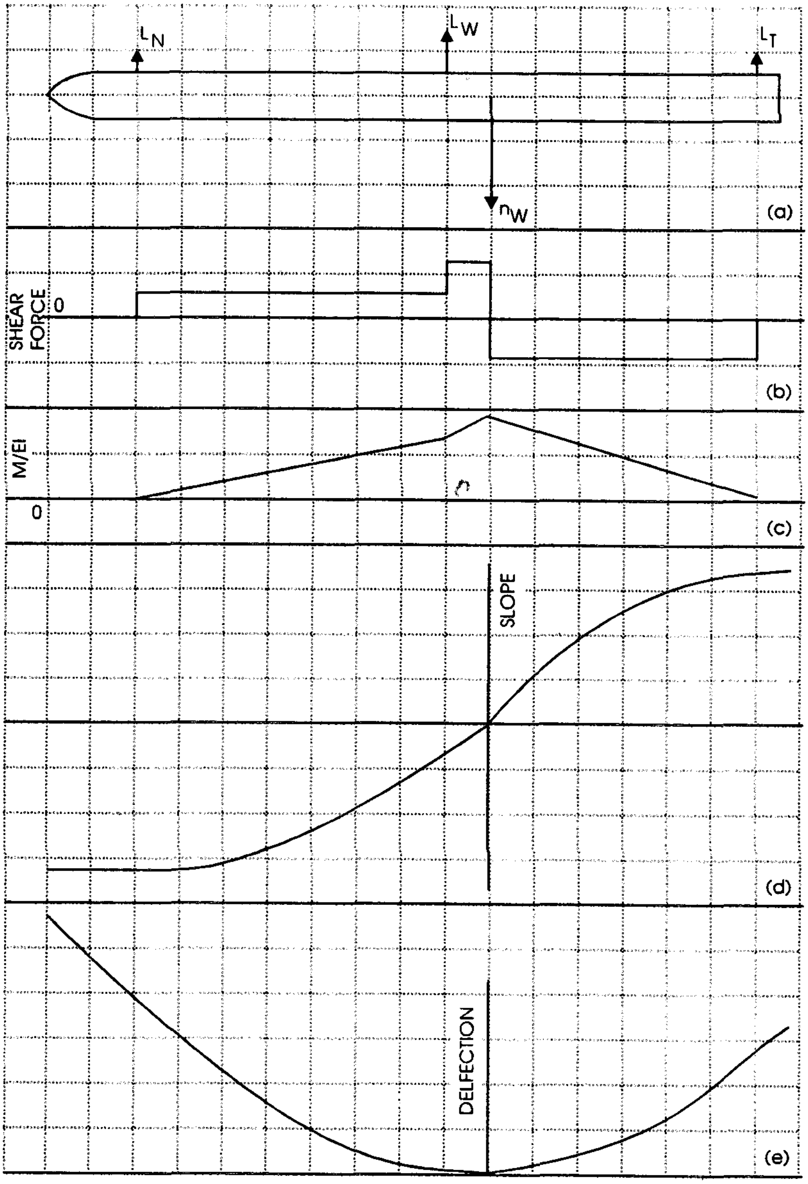


Figure 8-1. Loading and elastic curve beam. (a) Load diagram (b) Shear force diagram. (c) $\frac{M}{EI}$ curve (d) $\int \frac{M}{EI} dx$ slope diagram, (e) $\int dx \int \frac{M}{EI} dx$, deflection curve.

Example 1. Influence of the deflection on the guidance beam-width

The guidance beam is the heart of the guidance system. It serves to detect targets as they manoeuvre in three-dimensional space. The accuracy of detection is increased as the beam width is decreased. Usually, beam width of the order of 1° is used in modern weapon systems. However, as the beam width decreases, even small changes in the slope of the elastic curve can deflect the axis of the beam away from the target, with the consequent loss of information for tracking the target. The beam width will therefore constrain the allowable deflection and slope of the elastic curve at the location of the guidance seeker head. The distribution of $(E * I)$ along the length of the beam will have to be appropriately selected when the constraint on the allowable deflection and slope of the elastic curve has been specified at these locations.

The above conclusion is based only on considerations of the *static deflection* of the elastic curve. The dynamic behaviour of the structure and its influence on the functioning of the guidance subsystems located inside the configuration is also to be considered.

The seeker head is stabilised by a servo mechanism whose sole function is to isolate the seeker head from all disturbances (including those caused by the elastic deflection of the structure) and keep the guidance beam locked on to the target. However, the structure is also an elastic beam and therefore, it has its own structural vibration characteristics. The dynamic behaviour of the structure worsens the situation for the seeker head servo mechanism because of the periodicity associated with the structural disturbances. The band width of the servo mechanism will have to take into account the structural frequencies of the configuration also. A certain amount of isolation between the structural frequencies and the dynamic characteristics of the servo mechanism can be attempted in design. However, care needs to be taken in the selection of the servo frequency since, in the process the noise band width of the seeker-head servo also increases with increase in the structural natural frequencies. It is significant to note that the $(E * I)$ distribution that has been selected to control the static deflection of the elastic curve, also influences its structural frequencies. The structural frequency increases with increase in $(E * I)$.

If very little flexibility is available to the designer in the selection of the $(E * I)$ distribution, then the distribution of the manoeuvre lift on the configuration itself will have to be selected anew. Thus, in this phase of design, the structural designer will have to work closely with both the aerodynamic configuration designer and the designer of the seeker head servo.

Example 2. Thrust-axis vs elastic curve

In guided weapons, it is desirable to have the thrust axis collinear with the longitudinal axis of the configuration. This assures that the thrust force neither contributes to any disturbance moment about the centre of gravity of the configuration, nor to any out of plane lateral force on the configuration. However, since the structure is elastic, it bends under the influence of the lateral manoeuvre forces. It is therefore, inevitable that the deflection of the weapon will make the thrust axis inclined to the longitudinal axis of the configuration. This could result in a *force and moment bias* about the centre of gravity of the configuration. The bias moment and force can be minimised with the proper selection of the *EI* distribution function on configuration.

In weapon designs that use flexible nozzles for thrust vectoring, the *EI* distribution function (which has been selected from the point of view of the static deflection of the thrust axis) may often push the structural frequency close to the frequency of the servo system used in the thrust vectoring loop. This could result in deleterious coupling between the two subsystems.

The need for judicious choice of the (EI) distribution along the longitudinal axis of the configuration is illustrated by the above two examples. Usually, the specification of the preferred range in the value of μ to meet the propulsion requirement (discussed in Chapter 7), places a further constraint on the structural designer in realising the (EI) distribution in design. The maximum diameter of the configuration compatible with the preferred value of μ is fixed for a specified overall mass of the weapon. Under the circumstances, the designer has only two choices to meet the requirement of (EI) distribution:

(a) Choose the wall thickness of the structure of the configuration, while taking care to assure the requirement of the volume needed to accommodate the contents of the configuration.

(b) Choose the material of the structure if he has a constraint on the allowable wall thickness.

The interaction of the aerodynamic and control systems designs with the structural design of the configuration is brought out in the above paragraphs. The constraint that these systems place on the selection of the (EI) distribution along the axis of the configuration has been discussed. *The structural designer is expected to find a minimum weight solution for the structural design of the configuration while working under the above stated constraints.*

REFERENCE

1. Dixon, R. John (1980). Design engineering—inventiveness, analysis and decision making. Tata McGraw Hill Comany Ltd., New Delhi.

CHAPTER 9

INTERACTIONS IN GUIDED WEAPON SYSTEMS

It can be seen that guided weapon systems consist of a number of interconnected subsystems. Each of the subsystems has specific functions but is dependent on other subsystems for its proper functioning. The laws governing the functioning of each of the subsystems could also be different. Further, while the harmonious functioning of each of the interacting subsystems is demanded by the weapon system to meet its overall objectives, each of the subsystems itself has a different set of performance objective to fulfil. Efficient design of the weapon system therefore, demands a thorough understanding of the dependencies between the various subsystems. A systems approach is therefore needed to identify the priority in tackling their conflicting requirements.

If weapon system is looked at as a series of feedback loops, at least three levels of interactions can be discerned between them. The objectives of the subsystems at each of these levels, the constraints under which these should perform and the output demanded by the other subsystems from each of these levels are depicted in Table 9-1.

The details of each level of the hierarchy and the specific objective fulfilled by each of the interacting subsystems are taken up below.

9.1 FIRST LEVEL — THE OUTER LOOP

The highest level in the hierarchy is the outer loop. This deals with the interaction of the chosen guidance policy with the short period dynamics of the weapon configuration. This is the highest level of interaction of the overall system. The output from this interaction is:

- The range in the guidance gain that would be needed to achieve the least miss distance of the weapon from the target.
- The short period dynamic characteristics of the weapon configuration that is compatible with the chosen guidance gain in assuring the minimum miss distance.

- The lateral acceleration that is demanded by the chosen guidance policy to define the trajectory that is needed to assure the minimum miss distance.
- The longitudinal acceleration for the weapon that is compatible with the chosen guidance policy in ensuring the required speed for the weapon.

9.2 SECOND LEVEL

The second level in the hierarchy consists of those specialist subsystems that are needed to fulfil the output of the first level interactions. Thus:

- The guidance subsystem that is needed to ensure the first level requirement of the guidance gain.
- An aerodynamic configuration that is needed to ensure two of the first level requirements; namely
  - (a) The short period dynamic characteristics of the configuration
  - (b) The lateral acceleration demanded by the guidance policy to define the trajectory of the weapon
- The propulsion subsystem that ensures the longitudinal acceleration demanded by the first level of the hierarchy.
- The warhead subsystem that ensures the desired degree of kill probability compatible with the miss distance achievable by the chosen guidance policy.

9.3 THIRD LEVEL

The third level in the hierarchy consists of those subsystems that do not directly fulfil the requirements arrived at from the first level of the hierarchy. These subsystems are nevertheless very important from the point of view of the design of the second level subsystems of the weapon system. These are:

- The structures subsystem that gives a physical shape to the aerodynamic configuration, and which is needed to house all the airborne subsystems.
- The control subsystem that executes the guidance commands.
- The power supply designed to fulfil the power requirements of all onboard subsystems.

Table 9.1- Weapon system as a series of feed-back loops

Level I.	**Objective**	**Process**	**Functional features required**	**Constraints**
Outer loop	Minimise miss distance of weapon for all possible target attack trajectories	Systems study on interaction of chosen guidance policy with the short-period dynamics of configuration	1. Guidance gain 2. Speed ratio. 3. Rise time. 4. Command updating time. 5. Longitudinal acceleration. 6. Lateral acceleration.	"Intelligence information on the performance characteristics of the target.
Level II				
Guidance subsystem	1. Minimise the miss distance. 2. Minimise the lateral acceleration requirement	Systems study on the influence of the chosen guidance policy on the miss distance, lateral acceleration,	Guidance gain	1. Limits on the "look-angle" of the detector in the seeker head. 2. Limits on the sensitivity of the guidance receiver
Warhead	Achieving acceptable SSKP from the warhead	System study for assessing the lethal radius achievable by the type of warhead compatible with the miss distance.	1. Warhead shape function 2. Type of explosive 3. Mass of warhead	1. Availability of the warhead technology 2. Miss distance achievable
Aerodynamic configuration	1 Achieving acceptable short-period frequency 2. Generation of the lateral acceleration.	Systems study on the influence of the lift distribution function on the short-period frequency and lateral acceleration	1. Static margin, 2. Slope of configuration lift-curve. 3. Lift distribution on nose, wing and tail locations	1. Maximum angle of attack of configuration 2. Maximum angle of control deflection 3. Permissible rise-time

Propulsion subsystem	Minimum weight solution for the weapon system	Systems study for determination of the chosen propulsion policy on the overall weight of system	1 Propellant mass ratio for each propulsion stage 2. Overall ratio of mass of weapon to the mass of warhead, μ	Achievable *Isp*
Level III				
Structural design	1. Arriving at the desired shape of the elastic curve. 2. Minimum weight solution for the structure of the weapon configuration.	Systems study of the influence of the EI distribution of the configuratio on the elastic curve	Desired EI distribution of the configuration	Deflection and slope allowable at certain locations of the configuration
Control Subsystem	Execute the guidance commands with the least expenditure of power requirement	Systems study of the interaction of all other subsystems on the requirement of control power	Overall power requirement	The band width of other subsystems.
Power supplies subsystem	To provide the required power for all on-board operations with least self weight	Systems study on the available electro-chemical processes to achieve high power per mass of power supply	Achieving high power density in order to minimise self weight, high shelf life	Availability of technology

Viewed as a hierarchy, it is noticed that weapon system design splits up into a series of well defined levels. Each level has its own precise laws under which it functions, well defined objectives for it to achieve, and well defined constraints under which it should achieve these objectives. It is possible therefore to define the optimum level of performance one can expect from each level. *A high level of clarity is therefore possible in problem definition.* Since the performance levels desired from the subsystems at each level is definable uniquely, a yardstick is available to evaluate the performance at subsystem level. Thus, *systems study* has many useful features to recommend its use in the design of multidisciplinary systems.

Appendix A

ILLUSTRATIVE EXAMPLE—BEAM RIDER POLICY

As an illustration of the methodology of systems study developed in Chapter 4, the technique of systems study is applied now to determine the system functional characteristics needed for the design of an area defence weapon to counter attacks from low-flying aircraft. Area defence weapons are tactical guided weapons designed to defend a specified area around its launch position. The guidance policy selected for the weapon, in this example, is the basic beam rider policy. The operational requirements and the *intelligence* information gathered about *the target are given* below.

TARGET CHARACTERISTICS

Altitude of flight of the target	*50 m*
Speed of flight of the target	*300 m/s*
Farthest detection range of the target	*5000 m*
The maximum manoeuvre capability of target	*5 g*
Range of target when weapon takes off	2600 m for a 8 s reaction time
Radius of the defended area	1000 m from the launch site

The defence strategy would require the possibility of a second weapon being launched against the target, *within two seconds* after the launch of the first weapon.

SYSTEM SIMULATION STUDY

The essential functional features that are required to be determined by an analysis of the results of system simulation study are:

- The guidance gain,
- The ratio of the speed of the weapon to that of the target,
- The rise time of the weapon in its short period mode,
- The frequency at which the commands need to be updated,
- *The longitudinal acceleration needed for the propulsion of the* weapon, and

- The lateral acceleration demanded on the weapon by the chosen guidance policy.

Out of the above six functional features, the first task of the systems designer is the determination of the longitudinal *launch acceleration* for the weapon. The consideration on the choice of the launch acceleration is taken up for discussion now.

CHOICE OF LAUNCH ACCELERATION

The choice of the launch acceleration of the weapon is based on considerations of how far from the launch point should an oncoming target be intercepted by the weapon. This decision is dependent on the extent of the area around the launch site that is required to be defended. The inputs needed for this decision are:

- The relative speed of the weapon with respect to that of the oncoming target,
- The reaction time of the weapon system, and
- The farthest range from the launch site at which the target can be detected, unambiguously.

The influence of the above three parameters on the choice of the launch acceleration for the weapon are taken up for discussions now.

Unambiguous Target Detection Range

Usually, radar devices are used to detect the oncoming target. Since these devices work on the *echo* principle, these systems have all the limitation inherent to devices using this principle. One of the main handicaps of using radar devices is the difficulty associated with the identification of the target, in the presence of reflections from its surroundings. This is referred to as the problem of *clutter.* Detecting the target in the presence of clutter is particularly acute when the radar cross-section (RCS) of the target is very small compared to the reflecting area of the background. Depending upon the type of target and also on the sensitivity of the detector in the radar, there is a maximum range at which a target of a specified RCS can be detected unambiguously.

Reaction Time

After detecting the presence of the target unambiguously, a finite time is needed for the completion of the following activities, before the weapon leaves the launcher to intercept the target:

(a) Identification of the target, with a view to ascertain if the target is a friend or a foe, and

(b) To ascertain its heading, before assigning the task of intercepting it to a specified weapon battery.

Thereafter, a finite time is needed, at the weapon battery, for

(a) The acquisition of the target, and

(b) To assign the target to a specified weapon launcher.

Thereafter, a finite time is needed at the designated weapon launcher to

(a) Slew the launcher in the direction of the oncoming target, and

(b) To Prepare the weapon for launching towards the target.

The time taken for all these foregoing activities is known as the *reaction time* of the system. It is obvious that the greater the reaction time, the closer will the target be to the defended area.

It is apparent that during the time taken for the weapon system to react to the presence of the target, the target will be nearing the defended zone. Under the circumstances, the only way to intercept the target *beyond the defended zone* is to quickly accelerate the weapon. The higher will be the required launch-acceleration, if either the reaction time is higher, or the extent of the defended area is greater.

The point of interception of the weapon with the target will vary, depending upon its launch acceleration, the duration under launch acceleration, the speed at which the weapon is to be sustained, if required, after the completion of launch acceleration and on the reaction time of the weapon system. A nomogram is drawn in Fig. A-1, representing this dependency.

The distance covered by the target towards the defended area is a linear function of the reaction time. The distance covered by the weapon under the boost-acceleration is, however, a parabolic function of the boosting time. The subsequent distance covered by the weapon, during its sustained flight, is a linear function of the elapsed time and directed along the tangent to the parabola at the end of boosting time. From this nomogram, the interception points have been read off for different reaction times and tabulated in Table A-1.

It is seen from Table A-1 that it is *possible to defend* an area up to 1000 m from the launch point, *even with a reaction time of 8 seconds*. For this purpose, the weapon needs to be accelerated for 3 seconds with a launch acceleration of 100 m/s^2, and its speed

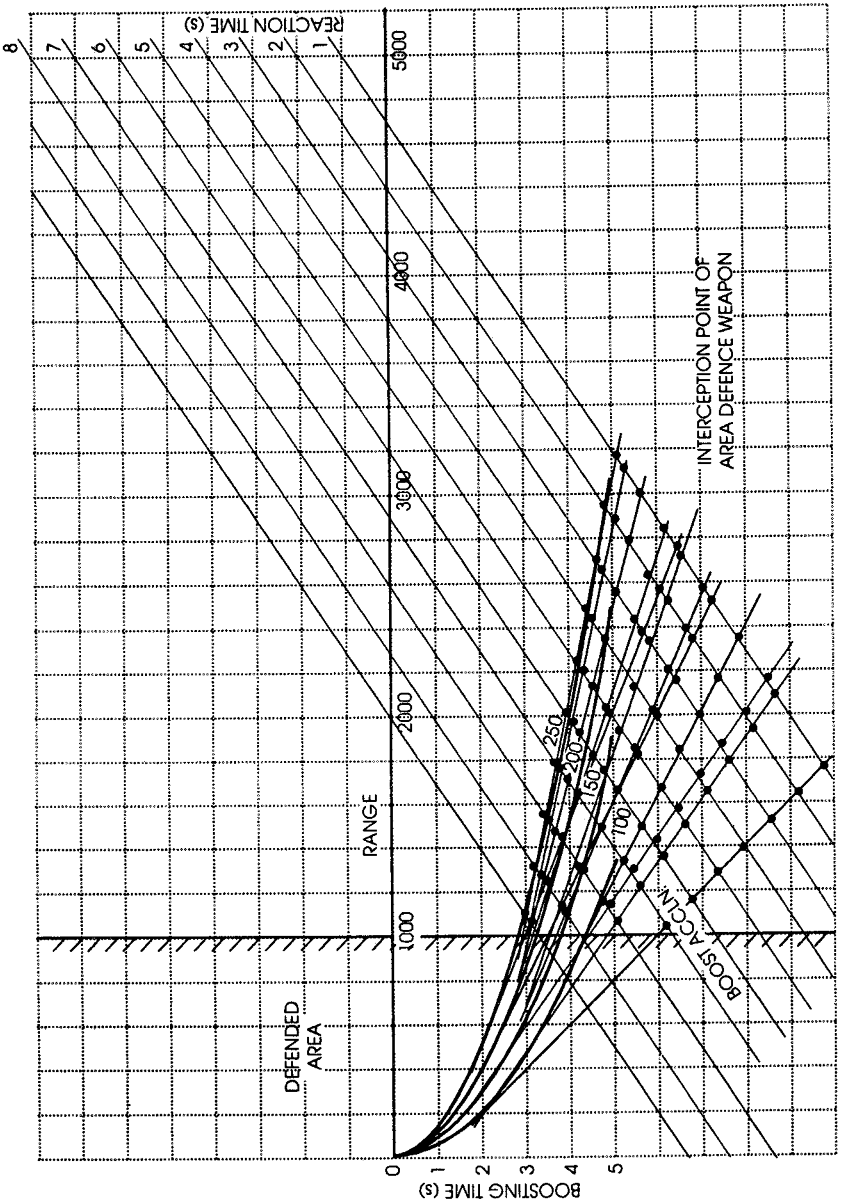


Figure A-1. Interception point of area defence weapon

needs to be sustained till interception at the boost-end speed of 300 m/s. The point of interception is 1075 m from the launch point. However, even if the boosting period were to be increased up to 5 s, there is *no possibility for a second launching* of a weapon against the same target.

A second launching however, is possible within one second after the launching of the first weapon, if the weapon were to be accelerated either for 2 s or for a higher duration at a launch acceleration ≥ 150 m/s^2. Thereafter, it is necessary to sustain the weapon at a speed ≥ 300 m/s till interception.

Table A-1. Interception range as a function of reaction time and boost acceleration

Launch acceleration: 100 m/s^2

Boosting	*Sustained*	*Range at interception (m) for a reaction time of:*						
time (s)	*speed m/s*	*4 s*	*5 s*	*6 s*	*7 s*	*8 s*	*9 s*	*10 s*
1	100	932.4	837.4					
2	200	1400.0	1280.0	1160.0	1040.0	*920.0*		
3	300	1675.0	1525.0	1375.0	1225.0	*1075.0*	925.0	
4	400	1828.6	1657.2	1485.8	1314.3	*1142.9*	968.6	
5	nil					*1155.4*	968.68	

Launch acceleration: 150 m/s^2

Boosting	*Sustained*	*Range at interception (m) for a reaction time of:*						
time (s)	*speed m/s*	*4 s*	*5 s*	*6 s*	*7 s*	*8 s*	*9 s*	*10 s*
1	150	1216.6	1100.0	1000.0	916.6			
2	300	1750.0	1600.0	1450.0	1300.0	*1150.0*	*1000.0*	850.0
3	450	2010.0	1830.0	1650.0	1470.0	*1290.0*	*1110.0*	930.0
4	600	2133.4	1933.4	1733.4	1533.4	*1353.4*	*1136.0*	944.0
5	nil			1748.8	1533.4	*1353.4*	*1136.0*	944.0

Launch acceleration: 200 m/s^2

Boosting	*Sustained*	*Range at interception (m) for a reaction time of:*						
time (s)	*speed m/s*	*4 s*	*5 s*	*6 s*	*7 s*	*8 s*	*9 s*	*10 s*
1	200	1460.0	1340.0	1220.0	1100.0	980.0		
2	400	2000.0	1828.6	1657.2	1485.8	*1314.2*	*1142.8*	971.4
3	600	2233.4	2033.4	1833.4	1633.4	*1433.4*	*1233.4*	*1033.4*
4	800	2327.2	2116.0	1889.2	1670.8	*1459.4*	*1250.0*	*1033.4*
5	nil	2344.8	2115.0	1889.2	1670.8	*1459.4*	*1260.0*	*1033.4*

Launch acceleration: 250 m/s^2

Boosting time (s)	*Sustained speed m/s*	*Range at interception (m) for a reaction time of:* 4 s	5 s	6 s	7 s	8 s	9 s	10 s
1	*250*	1659.0	1525.0	1386.4	1250.0	*1113.6*	977.2	841.0
2	*500*	2187.6	2000.0	1812.6	1625.0	*1437.6*	*1250.0*	*1062.4*
3	*750*	2392.8	2178.6	1964.2	1750.0	*1535.8*	*1321.4*	*1107.0*
4	1000	2461.6	2230.8	2000.0	1776.0	1552.2	1328.4	1107.0
5	nil	2461.6	2230.8	2000.0	1776.0	1552.2	1328.4	1107.0

The user's requirement of the capability to launch a second weapon within 2 s is possible however, if the launch acceleration was higher than 200 m/s^2 followed by sustaining the weapon at a speed greater than 500 m/s.

It is advisable to aim at the possibility of a greater time gap between successive launches, since this affords greater duration to assess the need for the second launch of the weapon. One way to achieve this aim would be to train the crew members operating the weapon battery with the view to reducing their reaction time.

Thus, based on launch acceleration considerations, two out of the six system functional features needed for the design of the weapon system can be determined. These are:

- Sustained speed of weapon $\geq 500\ m/s$, leading to the ratio of the speed of the weapon to that of the target,

$$\frac{Velocity\ of\ weapon}{Velocity\ of\ target} \geq 1.66$$

- The longitudinal acceleration needed for launching the weapon, $\geq 200\ m/s^2$ leading to boosting time ≥ 2.5 s.

After determining the launch acceleration, the other system functional features that are required to be determined solely, based on system simulation studies are:

- The guidance gain,
- The rise time of the weapon in its short period mode,
- The frequency at which the commands need to be updated, and
- The lateral acceleration demanded on the weapon by the chosen guidance policy.

The influence of the above system functional features on the final miss distance achieved by the weapon have been determined by system simulation studies. The results are discussed below.

RESULTS OF SYSTEM SIMULATION STUDIES

Crossing targets are those targets which do not have their velocity vector lying in the initially defined vertical reference plane XOZ. In fact, since their velocity vector has a finite azimuth angle $\Phi_{VT} \neq 0^0$, the trajectory of the target will lie out of the initially defined vertical plane. In the basic beam rider policy, the weapon is launched along the guidance beam, which lies initially on the reference XOZ plane. Therefore, the initial part of the trajectory of the weapon will also lie in the XOZ plane. Subsequently, the guidance policy would demand lateral accelerations on the weapon which would take it out of the XOZ plane.

Factors influencing Maximum Demanded Lateral Acceleration

1. *Maximum Demanded Lateral Acceleration as a Function of* Φ_{VT} *and Guidance Gain*

The effect of variations of Φ_{VT} on the maximum demanded lateral acceleration is plotted in Fig. A-2, as a function of guidance gain. In this plot the speed ratio is kept constant at 2.5. The results are tabulated in Table A-2.

Table A-2. Maximum demanded lateral acceleration as a function of ϕ_{VT} and guidance gain-beam rider guidance policy

Range = 2600 m, Speed ratio = 2.5, Weave-mode damping factor 0.7

Gain (m/s²/m)	Φ_{VT}=0°	10°	20°	30°	40°	50°	60°	70°
10.0	18.56	80.00	146.67	195.30	220.67	222.73	221.17	229.55
20.0	18.69	81.70	149.61	198.96	224.42	226.02	214.07	225.46
30.0	18.82	82.25	150.43	199.73	224.88	226.12	210.97	223.53
40.0	18.85	82.40	150.68	199.95	224.99	226.12	209.13	222.35

It is seen that at a constant value of the speed ratio,

- The maximum lateral acceleration is dependent only on Φ_{VT},

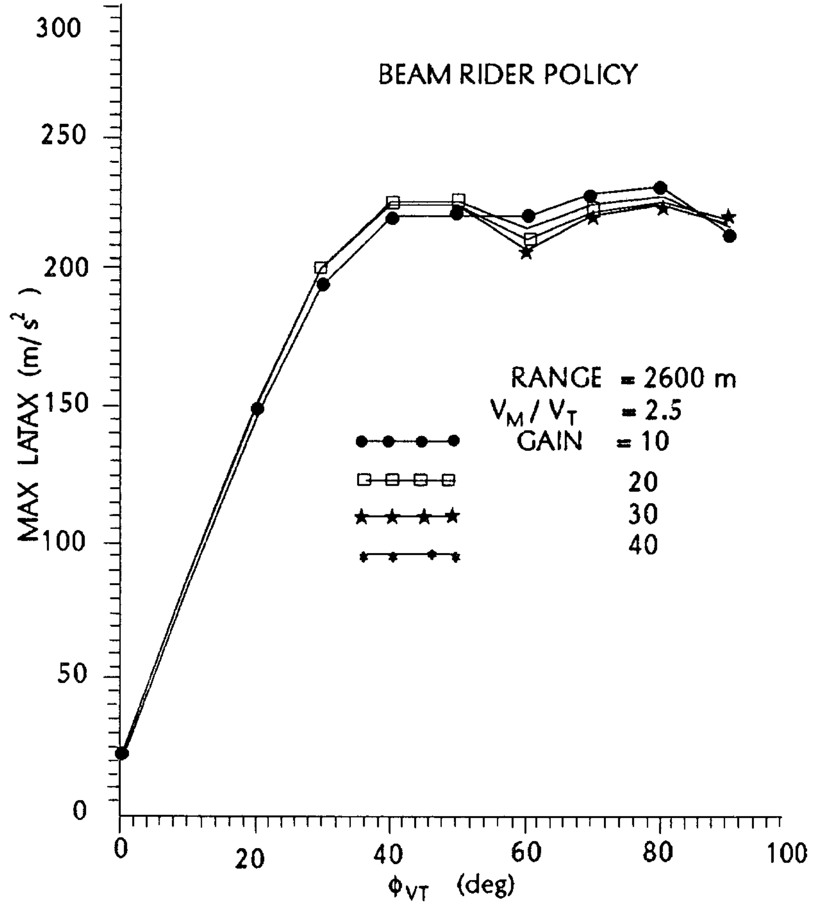


Figure A-2. Effect of Φ_{VT} and gain on maximum latax

- The lateral acceleration increases with increase in Φ_{VT}, *and reaches a peak value around* Φ_{VT} *=40°,*
- Variations in the *guidance gain has no influence* on the maximum demanded value of the lateral acceleration.

2. *Maximum Demanded Lateral Acceleration as a Function of* Φ_{VT} *and Speed Ratio*

The effect of Φ_{VT} and the speed ratio on the maximum demanded value of the lateral acceleration is plotted in Fig. A-3 keeping the guidance gain at a constant value of 20 m/s²/m. The results are also tabulated in Table A-3.

Table A-3. Maximum demanded lateral acceleration as a function of ϕ_{VT} and speed ratio-beam rider policy

Range = 2600 m, Gain = 20 $m/s^2/m$, Weave-mode damping factor = 0.7

Speed ratio	Φ_{VT}=0°	10°	20°	30°	40°	50°	60°	70°
1.5	18.86	81.85	145.64	182.02	182.01	149.75	128.86	135.65
2.0	18.62	80.26	144.98	188.16	203.90	193.41	171.43	180.53
2.5	18.69	81.70	149.61	198.96	224.42	226.02	214.07	225.46
3.0	19.18	84.43	156.14	211.03	243.97	254.29	256.74	270.43

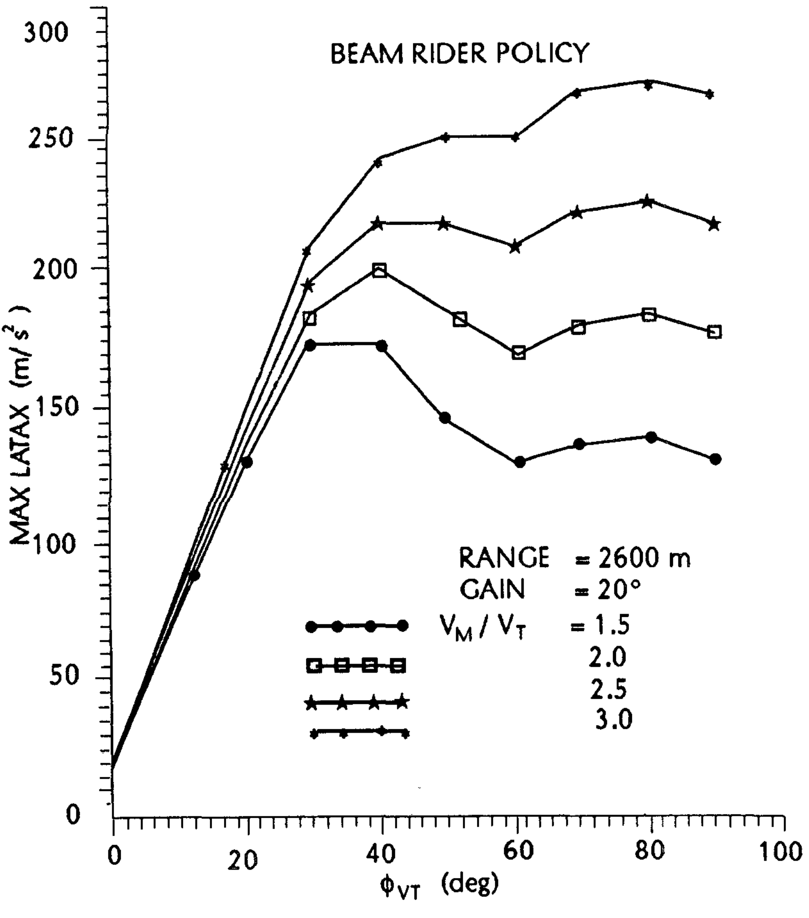


Figure A-3. Effect of Φ_{VT} and V_M / V_T on maximum latax

At a constant value of guidance gain,

(a) The maximum demanded *lateral acceleration increases with increase in* Φ_{VT}, reaches a maximum around 40°, and then decreases.

(b) The pattern of variation of the lateral acceleration is *skew symmetric* with respect to Φ_{VT}.

(c) The peak value of the demanded lateral acceleration shifts towards higher values of Φ_{VT} as the speed ratio increases from 1.5 to 3.0.

(d) At values of $\Phi_{VT} < 20°$, the maximum demanded value of the lateral acceleration is least affected by variations in the speed ratio.

3. *Maximum Demanded Lateral Acceleration as a Function of Guidance Gain and Velocity Ratio*

The effect of the combined variations of the guidance gain and the speed ratio on the maximum demanded value of lateral acceleration has been plotted in Fig. A-4 at a constant value of $\Phi_{VT} = 40°$. The results are also tabulated in Table A-4.

Table A-4. Maximum demanded lateral acceleration as a function of the guidance gain and the velocity ratio—beam rider policy

Range = 2600 m, $\Phi_{VT} = 40°$, Weave-mode damping factor = 0.

Speed ratio	*Gain =10 (m/s²/m)*	*Gain = 20 (m/s²/m)*	*Gain = 30 (m/s²/m)*	*Gain = 40 (m/s²/m)*
1.5	181.89	182.01	182.06	182.07
2.0	202.35	203.90	204.03	204.13
2.5	220.67	224.42	224.88	224.99
3.0	240.42	243.97	245.62	245.90

At constant values of Φ_{VT},

(a) The maximum demanded value of the lateral acceleration is dependent on the speed ratio only; *higher the speed ratio, the higher is the value of the maximum demanded lateral acceleration.*

(b) Lateral acceleration is independent of variations in the guidance gain.

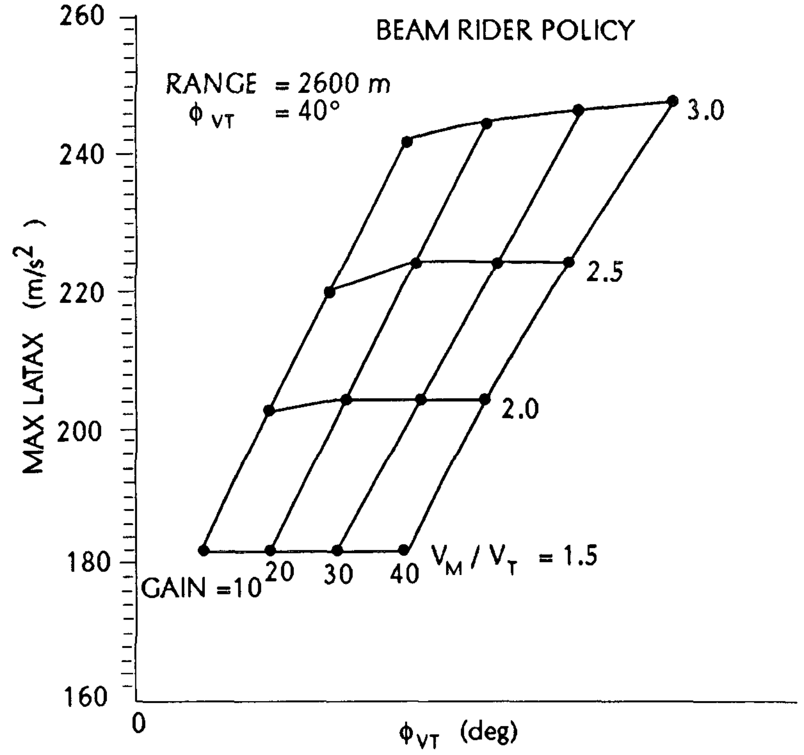


Figure A-4. Effect of V_M/V_T and gain on maximum latax

Factors Influencing the Miss Distance

1. *Miss Distance as a Function of* Φ_{VT} *and Guidance Gain*

The dependence of the miss distance as a function of Φ_{VT} and guidance gain is tabulated in Table A-5. It is also plotted in Fig. A-5, for a constant speed ratio of 2.5.

Table A-5. Miss distance as a function of Φ_{VT} and guidance gain

Range = 2600 m, Speed ratio = 2.5, Weave-mode damping factor = 0.7

Gain (m/s²/m)	Φ_{VT}=*0°*	*10°*	*20°*	*30°*	*40°*	*50°*	*60°*	*70°*
10	2.25	8.61	15.83	21.42	24.74	25.63	24.37	21.60
20	1.03	4.14	7.63	10.29	11.83	12.20	11.54	10.91
30	0.67	2.75	5.07	6.81	7.81	8.01	7.54	6.62
40	0.50	2.06	3.79	5.09	5.82	5.95	5.58	4.90

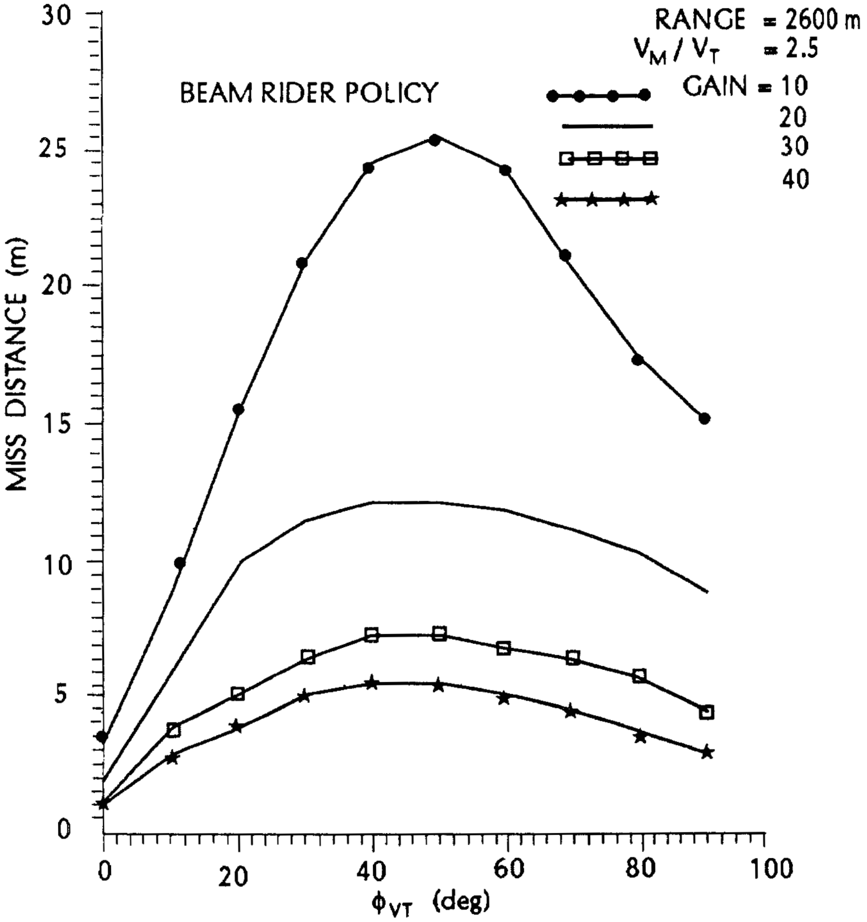


Figure A-5. Effect of Φ_{VT} and gain on miss distance

It is observed that at constant values of the speed ratio,

(a) The miss distance is almost symmetric with respect to Φ_{VT}.

(b) *At all values of guidance gain,* the miss distance increases as Φ_{VT} increases, reaches its maximum value around Φ_{VT} = 50° and then falls off gradually.

(c) The miss distance decreases with increase in the value of the guidance gain.

2. *Miss Distance as a Function of Φ_{VT} and Speed Ratio*

The dependence of the miss distance on Φ_{VT} and speed ratio is tabulated in Table A-6. It has also been plotted in Fig. A-6 keeping the guidance gain constant at 20 $m/s^2/m$:

- The curve is slightly skewed with the peak value of miss distance shifting towards higher values of Φ_{VT} as the speed ratio increases.

Table A-6. Miss distance as a function of ϕ_{VT} and speed ratio

Range = 2600 m, Gain = 20 $m/s^2/m$, Weave-mode damping factor = 0.7

Speed ratio	Φ_{VT}=0°	10°	20°	30°	40°	50°	60°	70°
1.5	0.97	3.86	6.94	8.89	9.23	7.91	5.61	3.51
2.0	0.99	3.93	7.15	9.45	10.51	10.28	9.04	7.29
2.5	1.03	4.14	7.63	10.29	11.83	12.20	11.54	10.19
3.0	1.09	4.46	8.28	11.31	13.27	14.08	13.84	12.80

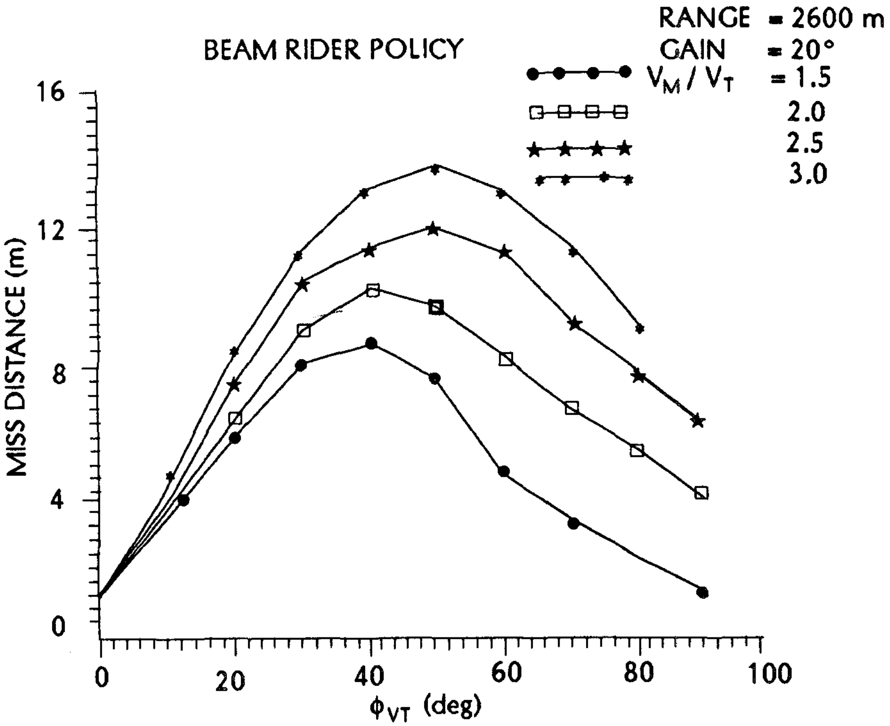


Figure A-6. Effect of Φ_{VT} and V_M/V_T on miss distance

3. *Effect of Guidance Gain and Velocity Ratio on Miss Distance*

The effect of the combined variation of the guidance gain and the speed ratio on the miss distance has been tabulated in Table A-7. The results are also plotted in Fig. A-7 for a constant value of $\Phi_{VT} = 40°$

Table A-7. Effect of guidance gain and velocity ratio on miss distance

Range = 2600 m, Φ_{VT} = 40°, Weave-mode damping factor = 0.7

Speed ratio	*Gain =10 ($m/s^2/m$)*	*Gain = 20 ($m/s^2/m$)*	*Gain = 30 ($m/s^2/m$)*	*Gain = 40 ($m/s^2/m$)*
1.5	18.60	9.23	6.14	4.60
2.0	21.39	10.51	6.96	5.20
2.5	24.74	11.83	7.81	5.82
3.0	28.90	13.27	8.69	6.46

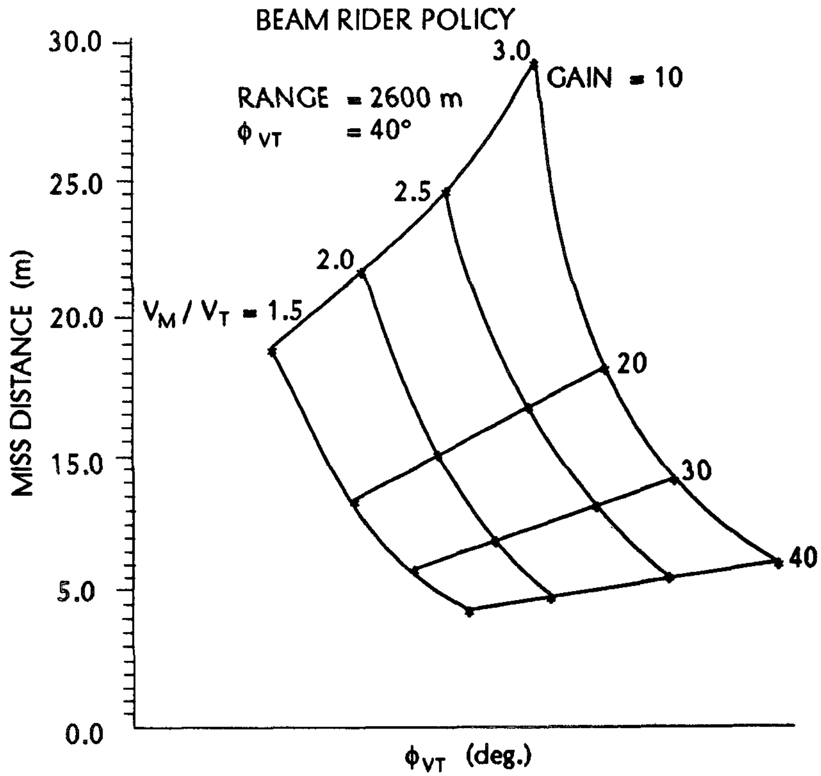


Figure A-7. Effect of V_M/V_T and gain on miss distance

It is observed that at constant values of Φ_{VT},

(a) *Increase in the guidance gain*, keeping the velocity ratio constant, *decreases the miss distance*. This pattern repeats itself at all speed ratio up to 3.

(b) The miss distance drastically falls off as the gain is increased from 10 m/s^2/m to 20 m/s^2/m.

(c) As the gain is further increased from 20 m/s^2/m to 40 m/s^2/m, the fall in the miss distance is only gradual.

(d) Of the two parameters influencing the miss distance while increase in the guidance gain minimises the miss distance, increase in the speed ratio increases the miss distance.

(e) *However, guidance gain has a dominant influence on the miss distance*. It is therefore recommended to choose guidance gain greater than 20 m/s^2/m, to achieve miss distance less than 10 m at speed ratios greater than 2.0.

(f) The speed ratio is however selected based on the requirement that the interception with the target should take place beyond the zone that needs to be defended.

4. *Effect of Command Updating Time and Rise Time on Miss Distance*

Table A-8. Effect of response time and command updating time on miss distance — beam rider policy

	Miss distance (m) achieved in a rise time, in seconds, of:									
t_{rise} ⇒	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
t_{update} ⇓										
0.01	13.33	13.35	13.37	13.39	13.40	13.42	13.44	13.46	13.48	13.50
0.02	13.37	13.39	13.41	13.43	13.45	13.47	13.49	13.51	13.53	13.55
0.03	13.41	13.43	13.45	13.47	13.49	13.51	13.53	13.55	13.57	13.59
0.04	13.45	13.47	13.49	13.51	13.53	13.55	13.57	13.59	13.61	13.64
0.05	13.49	13.51	13.53	13.55	13.57	13.59	13.62	13.64	13.66	13.68
0.06	13.53	13.55	13.57	13.59	13.62	13.64	13.66	13.68	13.70	13.72
0.07	13.57	13.60	13.62	13.64	13.66	13.68	13.70	13.72	13.74	13.75
0.08	13.62	13.64	13.66	13.68	13.70	13.72	13.74	13.75	13.77	13.77
0.09	13.67	13.69	13.71	13.73	13.74	13.76	13.77	13.78	13.78	13.78
0.10	13.71	13.73	13.75	13.77	13.78	13.79	13.80	13.80	13.79	13.77

The effect of the command updating time and the rise time on the miss distance achieved by the weapon is tabulated in Table A-8. It is noticed that

(a) As long as the updating time does not exceed 0.1 s, the miss distance remains practically constant around 13 m for all values of rise time not exceeding 0.1 s.

(b) As regards the choice of updating time, it is advisable to avoid a strong coupling between the two modes by selecting the updating time, well separated from rise time.

(c) Rise time of the order of 40 ms together with command updating interval of 100 ms are recommended.

5. *Effect of Command Updating Time and Rise Time on Maximum Lateral Acceleration Demanded on Weapon*

The effect of command updating time and the rise time on the maximum lateral acceleration demanded on the weapon is tabulated in Table A-9.

Table A-9. Effect of command updating time and the rise time on the maximum lateral acceleration demanded on the weapon

	Lateral acceleration (m/s²), achieved in a rise time, in seconds, of:									
$t_{rise}\rightarrow$	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
$t_{update}\downarrow$										
0.01	244.09	244.19	244.32	244.44	244.55	244.67	244.77	244.84	244.89	244.88
0.02	244.11	244.32	244.42	244.52	244.62	244.72	244.79	244.84	244.83	244.76
0.03	244.07	244.18	244.28	244.48	244.62	244.75	244.83	244.85	244.76	244.56
0.04	244.31	244.17	244.47	244.64	244.75	244.83	244.86	244.82	244.55	244.26
0.05	244.04	244.66	244.44	244.65	244.79	244.87	244.87	244.77	244.39	245.98
0.06	243.08	244.76	244.02	244.37	244.61	244.77	244.81	244.66	244.25	249.88
0.07	244.57	244.88	244.64	245.00	245.00	244.92	244.74	244.35	248.08	253.10
0.08	243.41	244.58	244.45	244.25	244.57	244.74	244.69	244.76	251.50	257.75
0.09	245.96	·243.91	245.27	243.82	244.19	244.46	244.41	249.80	255.72	260.63
0.10	243.13	245.75	244.00	24.34	245.23	244.92	246.20	252.51	259.91	265.93

It is seen that there is very little variation in the demanded maximum lateral acceleration, in the range of the updating time and the rise time of the system. The maximum demanded lateral acceleration is 265 m/s^2.

CONCLUSION

Based on an analysis of the results of the computer simulation studies, the following conclusions can be arrived at:

- The guidance gain should be chosen as high as possible, since miss distance decreases with increase in guidance gain. However, care should be taken in its selection, since a high gain system is also prone to be more noisy.
- The speed ratio should be selected to meet just the minimum requirements of the interception point, since the maximum demanded lateral acceleration increases with increase in the speed ratio.
- The rise time of the system and the command updating time should be well separated, to avoid any coupling between these two modes.

SYSTEMS SPECIFICATIONS

The following systems specifications are drawn out for the design of an area defence weapon, following a beam rider guidance policy, to meet the performance requirements specified by the user:

Guidance gain	$\geq 20\ m/s^2/m$
Ratio of the speed of the weapon to that of the target, $\frac{Velocity\ of\ weapon}{Velocity\ of\ target}$	$\geq$ 1.66
Sustained speed of weapon	$\geq 500\ m/s$
Rise time of the weapon in its short period mode.	$40\ ms$
Frequency at which the commands need to be updated	$\leq 0.1\ s$
Longitudinal acceleration needed for launching the weapon,	$\geq 200\ m/s^2$
Boosting time	$\leq 2.5\ s$
Longitudinal acceleration in the sustained phase of flight	$= 0.0$
Lateral acceleration demanded on the weapon by the chosen guidance policy	$\leq 245\ m/s^2$
Miss distance	$\leq 13\ m$

Appendix B

ILLUSTRATIVE EXAMPLE—CLOS POLICY

As an illustration of the methodology of systems study developed in Chapter 4, the technique is applied now to determine the system functional characteristics needed for the design of an area defence weapon to counter attacks from low-flying aircraft. Area defence weapons are tactical guided weapons designed to defend a specified area around its launch position. The guidance policy selected for the weapon, in this example, is the command-to-line of sight policy (CLOS) which is a modification of the basic beam rider policy discussed in Appendix A. The operational requirements and the *intelligence* information gathered about the target are given below.

TARGET CHARACTERISTICS

Altitude of flight of the target	*50 m*
Speed of flight of the target	*300 m/s*
Farthest detection range of the target	*5000 m*
Maximum manoeuvre capability of target	*5 g*
Range of target when weapon takes off	*2600 m for a 8 s reaction time*
Radius of the defended area.	*1000 m from the launch site*

The defence strategy would require the possibility of a second weapon being launched against the target, *within two seconds* after the launch of the first weapon.

SIMULATION STUDY

The essential functional features that are required to be determined by an analysis of the results of a simulation study are:

(i) the guidance gain,
(ii) the ratio of the speed of the weapon to that of the target,
(iii) the rise time of the weapon in its short period mode,
(iv) the frequency at which the commands need to be updated,
(v) the longitudinal acceleration needed for the propulsion of the weapon, and
(vi) the lateral acceleration demanded on the weapon by the chosen guidance policy.

Of the above six functional features, the first task of the systems designer is the determination of the longitudinal *launch acceleration* for the weapon. The consideration on the choice of the launch acceleration is discussed in Appendix A.

Based on the launch acceleration considerations, two of the six system functional features have been determined. These are:

Sustained speed of the weapon	$\geq$ 500 m/s leading to $\frac{Velocity\ of\ weapon}{Velocity\ of\ target} \geq 1.66$
Longitudinal acceleration needed for launching the weapon	$\geq 200\ m/s^2$
Boosting time	$\leq$ 2.5 s

After determining the launch acceleration, the other system functional features that are required to be determined based only on system simulation are: (a) the guidance gain, (b) the rise time of the weapon in its short period mode, (c) the frequency at which the commands need to be updated, (d) the lateral acceleration demanded on the weapon by the chosen guidance policy.

RESULTS OF SYSTEM SIMULATION STUDIES

The influence of the above system functional features on the final miss distance achieved by the weapon have to be assessed from the results of system simulation studies. The results are discussed below.

Factors Influencing Maximum Demanded Lateral Acceleration

1. *Maximum Demanded Lateral Acceleration as a function of* Φ_{VT} *and Guidance Gain—CLOS Policy*

The results are tabulated in Table B-1. The effect of variations of Φ_{VT} on the maximum demanded lateral acceleration is also plotted in Fig. B-1, as a function of guidance gain. In this plot, the speed ratio is kept constant at 2.5.

Table B-1. Maximum demanded lateral acceleration as a function of Φ_{VT} and guidance gain—CLOS policy

Range = 2600 m, Speed ratio = 2.5, Weave-mode damping factor, zeta = 0.7

Gain *(m/s²/m)*	Φ_{VT} *=0°*	*10°*	*20°*	*30°*	*40°*	*50°*	*60°*	*70°*
10.0	18.54	82.88	151.53	201.39	228.24	233.01	220.15	196.10
20.0	18.84	82.87	151.41	201.20	228.02	232.87	220.21	196.17
30.0	18.91	82.96	151.61	201.52	228.44	233.20	220.18	196.30
40.0	18.92	82.99	151.59	201.36	228.03	232.59	219.81	195.97

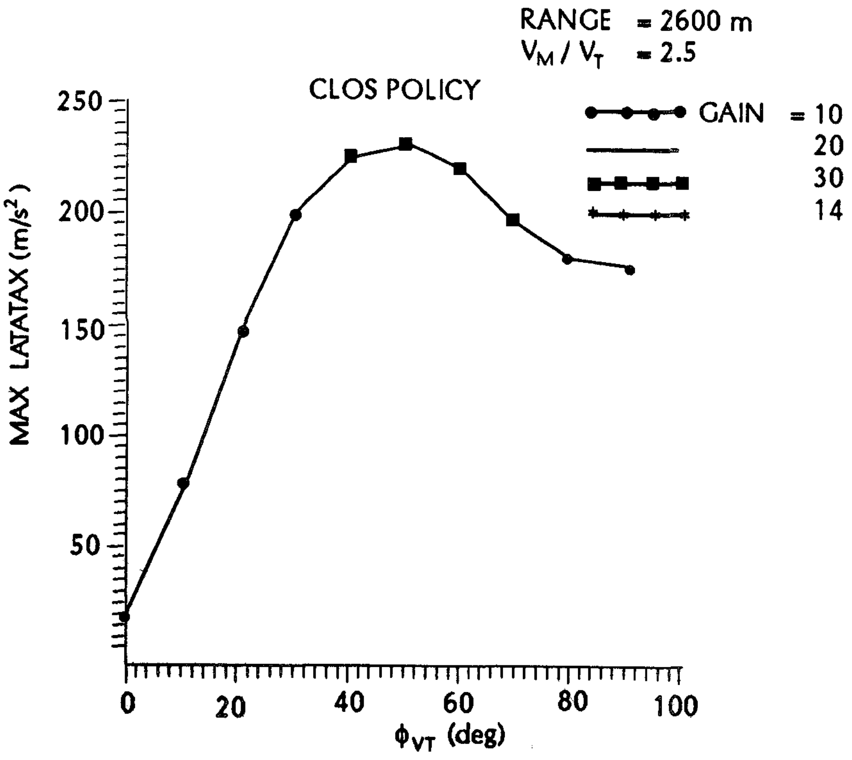


Figure B-1. Effect of Φ_{VT} and gain on maximum latax

(a) *The* maximum demanded lateral acceleration is dependent only on Φ_{VT}.

(b) *The lateral acceleration increases with increase in* Φ_{VT}, reaches a maximum around 50°, and then decreases.

(c) the maximum demanded lateral acceleration, at the same value of Φ_{VT}, is not influenced by variations in the value of the guidance gain.

(d) The pattern of the dependence of the maximum demanded lateral acceleration on Φ_{VT} is similar to what is noticed in the case of the basic beam rider policy.

2. *Maximum Demanded Lateral Acceleration as a Function of* Φ_{VT} *and Speed Ratio—CLOS Policy*

The effect of Φ_{VT} and the speed ratio on the maximum demanded value of the lateral acceleration is plotted in Fig. B-2, keeping the guidance gain at a constant value of 20 $m/s^2/m$. The results are also tabulated in Table B-2:

Table B-2. Maximum demanded lateral acceleration as a function of Φ_{VT} and speed ratio—CLOS policy

Range = 2600 m, Gain = 20 $m/s^2/m$, Weave-mode damping factor = 0.7

Speed ratio	$\Phi_{VT}=0°$	*10°*	*20°*	*30°*	*40°*	*50°*	*60°*	*70°*
1.5	19.02	82.74	144.93	179.59	184.93	167.74	138.52	117.98
2.0	18.75	81.20	146.09	189.39	207.50	203.05	182.41	157.05
2.5	18.84	82.87	151.41	201.20	228.02	232.87	220.21	196.17
3.0	19.05	86.14	159.09	214.85	248.85	260.99	254.51	234.54

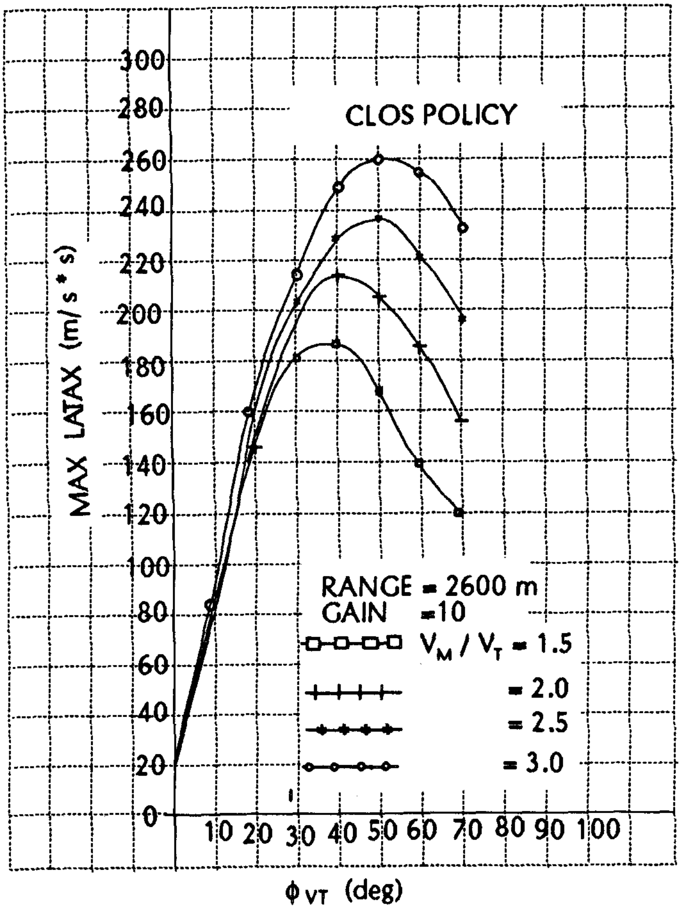


Figure B-2. Effect of Φ_{VT} and V_M/V_T on maximum latax

- For values of Φ_{VT} less than 20°, the maximum demanded value of the lateral acceleration has very little variation with variations in the speed ratio.
- The peak value of the demanded lateral acceleration shifts towards higher values of Φ_{VT} as the speed ratio increases from 1.5 to 3.0.

3. *Maximum Demanded Lateral Acceleration as a Function of Guidance Gain and Velocity Ratio—CLOS Policy*

The combined affect of variations in the guidance gain and the speed ratio on the maximum demanded value of lateral acceleration has been plotted in Fig. B-3, at a constant value of $\Phi_{VT} = 40°$. The results are also tabulated in Table B-3.

Table B-3. Maximum demanded lateral acceleration as a function of the guidance gain and the velocity ratio—CLOS policy

Range = 2600 m, Φ_{VT} = 40°, Weave-mode damping factor = 0.7

Speed ratio	*Gain =10 (m/s²/m)*	*Gain = 20 (m/s²/m)*	*Gain = 30 (m/s²/m)*	*Gain = 40 (m/s²/m)*
1.5	185.09	184.93	185.35	184.86
2.0	207.25	207.50	207.08	207.47
2.5	228.24	228.02	228.44	228.03
3.0	249.04	248.85	248.88	249.28

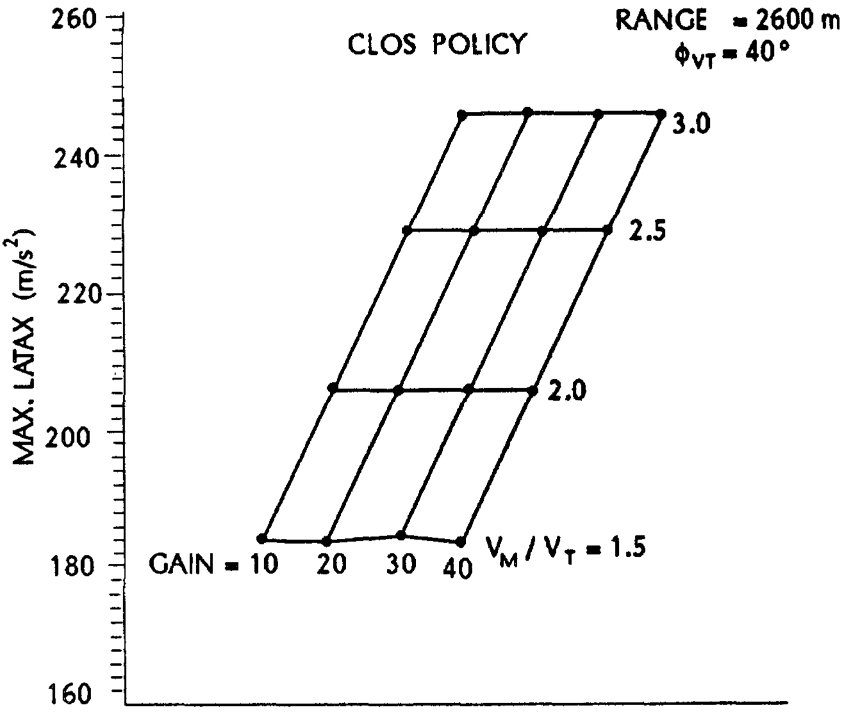


Figure B-3. Effect of V_M/V_T and gain on maximum latax

The pattern of behaviour is similar to that of the basic beam rider policy.

The maximum demanded value of the lateral acceleration is:

(a) *Dependent on the speed ratio only*; higher the speed ratio, higher is the lateral acceleration.

(b) It is independent of variations in the guidance gain.

Factors Influencing the Miss Distance

1. *Miss Distance as a Function of* Φ_{VT} *and Guidance Gain*

The dependence of the miss distance as a function of Φ_{VT} and guidance gain is tabulated in Table B-4. It is also plotted in Fig. B-4, for a constant speed ratio of 2.5.

Table B-4. Miss distance as a function of Φ_{VT} and guidance gain—CLOS policy

Range = 2600 m, Speed ratio = 2.5, Weave-mode damping factor = 0.7

Gain $(m/s^2/m)$	$\Phi_{VT}=0°$	*10°*	*20°*	*30°*	*40°*	*50°*	*60°*	*70°*
10	1.32	1.32	1.30	1.29	1.26	1.24	1.21	1.19
20	0.58	0.58	0.57	0.57	0.57	0.57	0.56	0.56
30	0,38	0.38	0.37	0.37	0.37	0.37	0.37	0.36
40	0.28	0.29	0.28	0.28	0.28	0.27	0.27	0.27

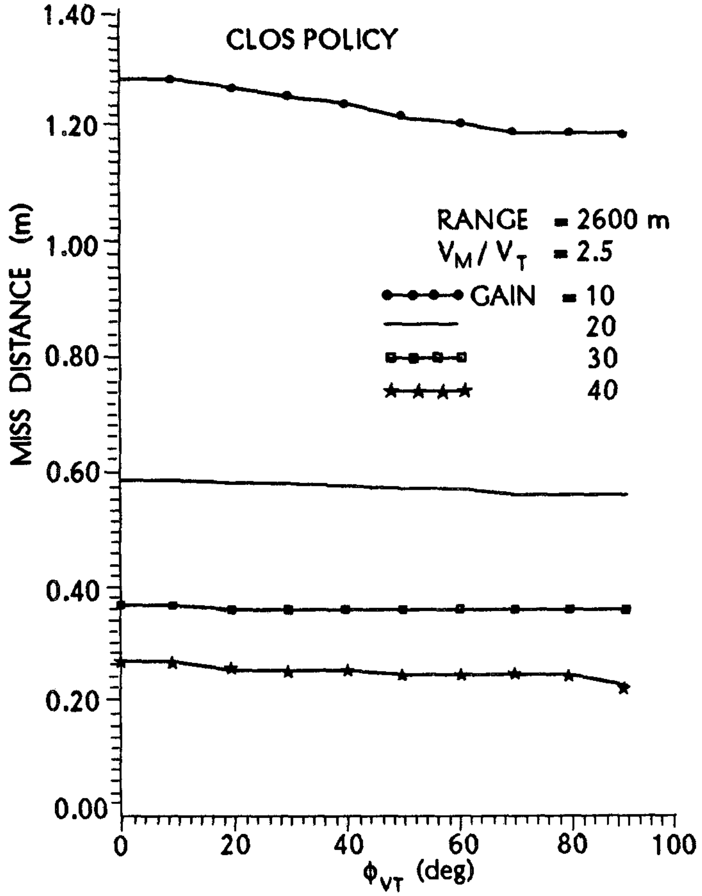


Figure B-4. Effect of Φ_{VT} and gain on miss distance

The pattern is in total contrast to that of the basic beam rider policy shown in Fig. A-5. The miss distance is:

(a) *Almost constant with respect to* Φ_{VT} at all values of guidance gain.

(b) *The miss distance decreases with increase in the value of the guidance gain.*

(c) The miss distance is less by an order of magnitude.

2. *Miss Distance as a Function of* Φ_{VT} *and Speed Ratio*

The dependence of the miss distance on Φ_{VT} and speed ratio is tabulated in Table B-5. It has also been plotted in Fig. B-5 keeping the guidance gain constant at 20 $m/s^2/m$. It is noticed that

Table B-5. Miss distance as a function of Φ_{VT} and speed ratio—CLOS policy

Range = 2600 m, Guidance gain = 20 m/s²/m, Weave-mode damping factor = 0.7.

Speed ratio	Φ_{VT} =0°	10°	20°	30°	40°	50°	60°	70°
1.5	0.55	0.55	0.55	0.54	0.54	0.54	0.53	0.53
2.0	0.56	0.56	0.56	0.56	0.55	0.55	0.55	0.54
2.5	0.58	0.58	0.57	0.57	0.57	0.57	0.56	0.56
3.0	0.60	0.59	0.59	0.59	0.58	0.58	0.57	0.57

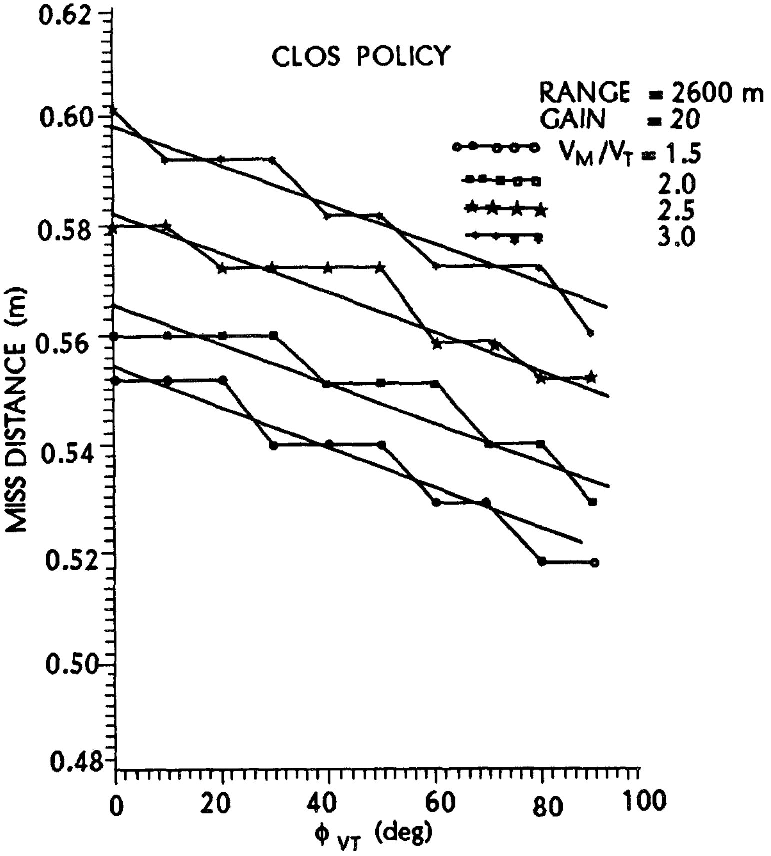


Figure B-5. Effect of Φ_{VT} and V_M/V_T on miss distance

It is seen that

(a) The pattern is in total contrast to that of the basic beam rider policy shown in Fig. A-6.

(b) Miss distance is a constant at all values of the speed ratio and Φ_{VT}, at a specified value of guidance gain. The variation in miss distance is of the order of 0.05m only.

3. *Effect of Guidance Gain and Velocity Ratio on Miss Distance*

The combined effect of the guidance gain and the speed, on the miss distance achieved by the weapon is tabulated in Table B-6.

Table B-6. Effect of guidance gain and velocity ratio on miss distance—CLOS policy.

Range = 2600 m, Φ_{VT} = 40°, Weave-mode damping factor = 0.7

Speed ratio	*Gain =10 (m/s²/m)*	*Gain = 20 (m/s²/m)*	*Gain = 30 (m/s²/m)*	*Gain = 40 (m/s²/m)*
1.5	1.13	0.54	0.35	0.26
2.0	1.18	0.55	0.36	0.27
2.5	1.26	0.57	0.37	0.28
3.0	1.38	0.58	0.37	0.28

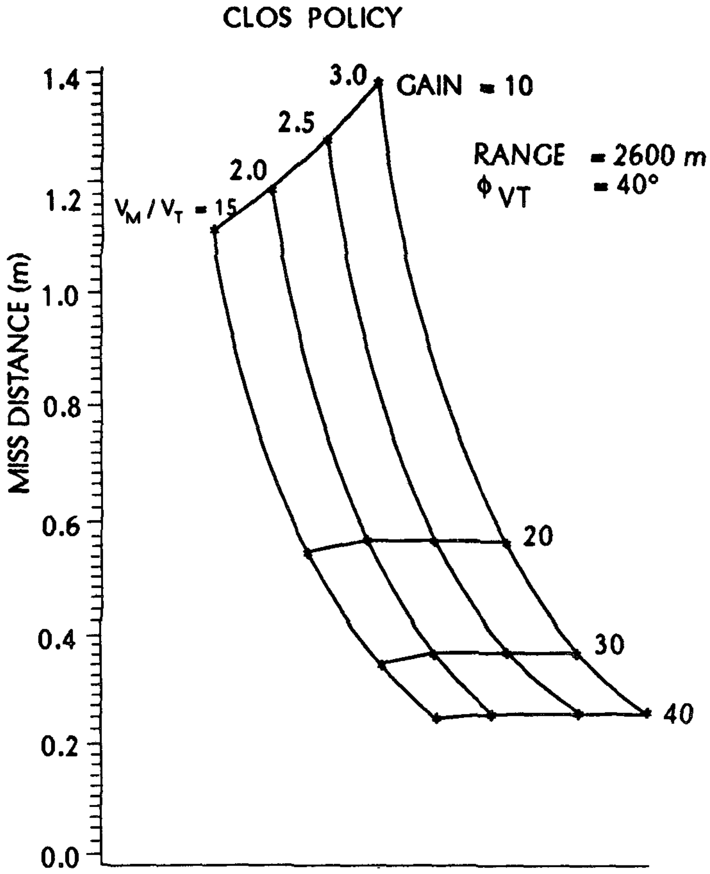


Figure B-6. Effect of V_M/V_T and gain on miss distance

The results are also plotted in Fig. B-6. The following observations are made:

- The pattern is similar to the basic beam rider shown in Fig. A-7.
- Guidance gain is the only factor influencing the miss distance—higher the guidance gain, the lower is the miss distance.
- The miss distance achieved by this policy is lower by an order of magnitude from what is attainable from the basic beam rider policy.

4. *Effect of Command Updating Time and Rise Time on Miss Distance*

The effect of the command updating time and the rise time, on the miss distance achieved by the weapon is tabulated in Table B-7. It is noticed that

Table B-7. Effect of response time and the command updating time on the miss distance—CLOS policy

	Miss distance (m), achieved in a rise time in seconds, of:									
$t_{rise} \Rightarrow$	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
$t_{update} \Downarrow$										
0.01	0.60	0.69	0.89	0.57	1.55	2.26	2.89	3.49	4.07	4.62
0.02	0.59	0.61	0.69	0.91	0.68	1.84	2.57	3.26	3.90	4.47
0.03	0.60	0.61	0.67	0.91	0.71	1.88	2.67	3.40	4.05	4.65
0.04	0.67	0.60	0.65	0.60	0.83	2.47	3.21	3.92	4.53	5.13
0.05	0.66	0.60	0.66	0.60	0.87	2.55	3.33	4.09	4.72	5.34
0.06	0.61	0.61	0.76	0.97	1.44	1.00	1.83	2.95	3.81	4.60
0.07	0.70	0.61	0.68	0.60	0.92	0.64	1.03	1.90	3.15	6.18
0.08	0.61	0.65	0.60	0.88	0.88	1.02	1.69	3.06	4.04	4.18
0.09	0.62	0.67	0.61	0.91	0.72	1.03	2.09	3.34	4.34	4.58
0.10	0.90	0.64	0.74	0.72	0.71	0.59	1.04	2.00	3.52	3.90

(a) As long as the updating time does not exceed 0.1 s, the miss distance remains practically constant around 1.0 m for all values of rise time not exceeding 0.04 s.

(b) As regards the choice of updating time, it is advisable to avoid a strong coupling between the two modes by selecting the updating time, well separated from rise time.

5. *Effect of Command Updating Time and Rise Time on Maximum Lateral Acceleration Demanded on Weapon*

The effect of the command updating time and the rise time, on the maximum lateral acceleration demanded on the weapon is tabulated in Table B-8. It is noticed that

Table B-8. Effect of response time and the command updating time on the maximum lateral acceleration demanded on the weapon—CLOS policy

Maximum lateral acceleration (m/s^2), demanded on the weapon for a rise time, in seconds, of:										
$t_{rise} \Rightarrow$	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
$t_{update} \Downarrow$										
0.01	251.2	253.1	257.1	254.3	279.7	293.4	306.1	318.1	328.8	337.9
0.02	248.8	250.8	252.0	256.5	257.8	284.3	298.8	312.6	324.9	334.8
0.03	248.8	250.5	251.3	256.0	257.7	285.0	300.7	315.4	327.6	337.9
0.04	243.3	245.3	246.2	250.9	252.3	297.0	311.9	326.2	337.3	346.8
0.05	241.8	244.7	247.1	249.9	253.1	298.2	314.1	329.2	340.7	350.3
0.06	248.3	251.8	251.8	257.5	275.5	256.7	282.1	304.8	321.4	335.6
0.07	241.1	244.4	247.3	248.9	253.0	246.6	255.2	281.8	307.0	364.1
0.08	245.5	247.2	249.3	252.7	255.9	255.5	278.2	306.3	325.3	325.8
0.09	246.3	247.9	249.4	253.2	253.6	256.2	286.8	311.9	330.7	333.1
0.10	241.3	240.3	240.7	250.8	247.2	244.5	254.2	283.0	313.5	319.3

(a) As long as the command updating time is not greater than 0.10 s the rise time needs to be limited to 0.06 s, if the maximum demanded lateral acceleration should not exceed 300 m/s^2.

(b) However, if rise time is limited to 0.04 s, then one can achieve miss distances less than 1 m without the maximum lateral acceleration exceeding 258 m/s^2.

CONCLUSION

Based on an analysis of the results of the computer simulation studies, the following conclusions can be arrived at:

- The guidance gain should be chosen as high as possible, since miss distance decreases with increase in guidance gain. However, care should be taken in its selection, since a high gain system is also prone to be more noisy.
- The speed ratio should be selected to meet just the minimum requirements of the interception point, since the maximum demanded lateral acceleration increases with increase in the speed ratio.
- The rise time of the system and the command updating time should be well separated, to avoid any coupling between these two modes.

SYSTEMS SPECIFICATIONS

The following systems specifications are drawn out for the design of an area defence weapon, following a beam rider guidance policy, to meet the performance requirements specified by the user:

Guidance gain	$\geq 20\ m/s^2/m$
Ratio of the speed of the weapon to that of the target, $\frac{Velocity\ of\ weapon}{Velocity\ of\ target}$	≥ 1.66
Sustained speed of weapon	$\geq 500\ m/s$
Rise time of the weapon in its short period mode	$40\ ms$
Frequency at which the commands need to be updated	$\leq 100\ ms$
Longitudinal acceleration needed for launching the weapon	$\geq 200\ m/s^2$
Boosting time	$\leq 2.5\ s$
Longitudinal acceleration in the sustained phase of flight	$= 0.0$
Lateral acceleration demanded on the weapon by the chosen guidance policy	$\leq 256\ m/s^2$
Miss distance	$\leq 0.6\ m$

Appendix C

SYSTEMS STUDY ON SHOULDER FIRED MISSILES

As an illustration of the methodology developed in Chapter 5 for the systems study on *proportional navigational policy*, the technique is applied now to identify the system functional characteristics needed for the design of a *shoulder fired missile*. These weapons are used for countering attacks from low-flying enemy aircraft. These are not *area defence weapons,* as discussed in Appendix B, but are essentially *infantry weapons,* fired from the shoulders of a frontline infantry soldier. Proportional navigation guidance schemes are used in these weapons.

In an operational situation, the infantry soldier rests the launch tube of the weapon on his shoulders, and trains the weapon against oncoming target. He waits for the target to come within the range of the weapon, and fires the weapon when the seeker head of the weapon locks on to the infra-red signals emanating from the target. Thereafter, the weapon homes towards the target following a proportional navigation trajectory.

The operational requirements and the *intelligence* information gathered about the target that are relevant for conducting systems study for the design of such a weapon are given below.

TARGET CHARACTERISTICS

Altitude of flight of the target	300 m
Speed of flight of the target	300 m/s
Farthest unambiguous detection range of target	5000 m

SYSTEM SIMULATION STUDY

The essential functional features that are required for the design of the weapon system have to be determined by an analysis of the results of system simulation study. The specific system functional features that have to be identified from such systems studies are: (a) the guidance gain, (b) the ratio of the speed of the weapon to that of the target, (c) the rise time of the weapon in its short period mode, (d) the frequency at which the commands need to be updated, (e) the longitudinal acceleration needed for the propulsion of the weapon, and (f) the lateral acceleration demanded on the weapon by the chosen guidance policy.

Out of the above six functional features, the first task of the systems designer is the determination of the longitudinal *launch acceleration* for the weapon. The considerations on the choice of the launch acceleration are taken up for discussion now.

In the example cited in Appendix B, the weapon was designed for the defence of *an extended area* around the launch point. The extent of the defended area from the launch point therefore sets the criteria for the choice of the level of the boost acceleration and the duration of the boost acceleration. The shoulder-fired weapons, on the other hand, are not meant to be used as an area defence weapon. All that is required of it is, that the weapon should be capable of intercepting the oncoming target, preferably head-on. The range at interception, therefore, is not a criterion for choosing the level of booster acceleration and the duration of the booster acceleration. Other considerations will therefore have to be thought of, for fixing the level of the boost acceleration and the duration of the boost acceleration. Without any loss of generality, the following three thrusting programmes are considered.

THRUSTING PROGRAMMES

Case 1. The weapon is continuously accelerated till interception with the target.

Case 2. The weapon is imparted a high level of acceleration for a short duration of time, during the boosting phase. This is followed by a sustained phase of flight. During this phase of flight, the boost-end speed is maintained essentially constant till interception with the target.

Case 3. The weapon is accelerated to a very high velocity in a short duration boosting phase. The weapon is then coasted till an interception with the target is achieved. During the coasting period, the boost-end momentum is expended for overcoming the decelerating forces due to the aerodynamic drag of its configuration.

Computer simulation studies on the weapon system based on the above three cases of thrusting programmes have been conducted extensively at DRDL[1]. The results from the study are presented below to illustrate how the thrusting programmes have been decided upon.

The infantry soldier leans the weapon on his shoulders and aims the weapon against the incoming target. He then locks the homing head of the weapon to the signals from the target and *launches the weapon against the target, giving the weapon a lead angle* in the direction of heading of the target. In this way, the initial trajectory of the weapon would approximate a constant bearing course.

The target, on the other hand, will head either towards the launch point or away from it, depending upon whether the azimuth angle of its initial velocity vector Φ_{V_T} is either 180° or 0°. Whenever the azimuth angle of the velocity vector of the target has a value other than either 180° or 0°, the target is said to be a crossing target. In such cases, the guidance policy would demand lateral accelerations on the weapon to take it outside the initially defined weapon-launching plane. The cases of crossing targets are therefore worth studying, since these give the requirement on the maximum lateral acceleration demanded on the weapon by such a policy.

CASE 1. WEAPON IS CONTINUOUSLY BOOSTED TILL INTERCEPTION WITH THE TARGET

Effect of Guidance Gain on Maximum Lateral Acceleration Demanded on Weapon

The simulation studies are conducted to assess the influence of variations of guidance gain and Φ_{V_T} on the maximum lateral acceleration demanded on the weapon. The studies are conducted at a constant launch acceleration of 200 m/s^2. The result of the study is plotted in Fig. C-1.

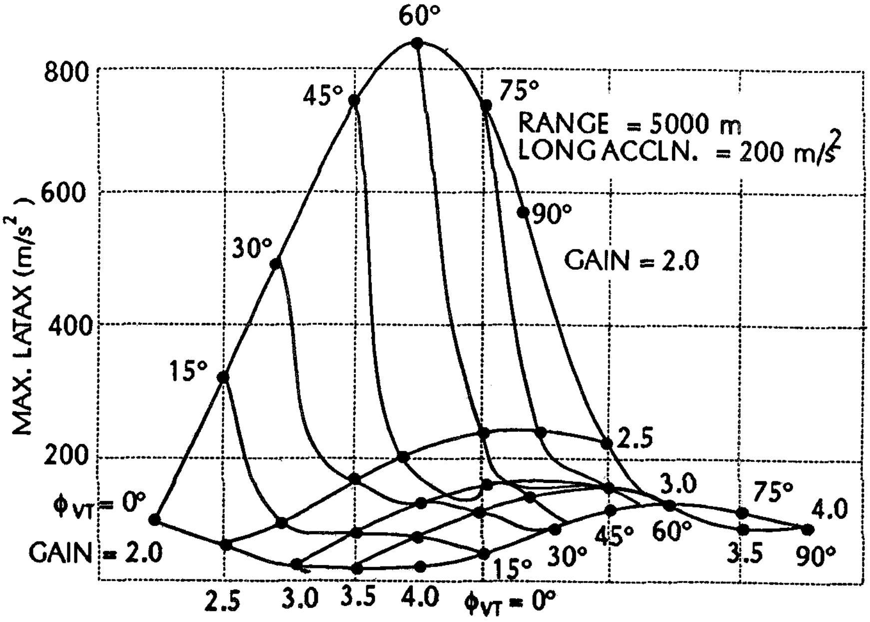


Figure C-1. Effect of Φ_{VT} and gain on maximum latax

The following observations are made:

At constant values of Φ_{V_T}*,*

- The lateral acceleration demanded on the weapon *falls off drastically as the guidance gain is increased from 2 to 2.5*
- Thereafter, the lateral acceleration, shows a slightly decreasing tendency with increase in the values of the guidance gain.

Till a value of Φ_{V_T} *equal to 60° is reached,*

- There is an increase in the maximum demanded lateral acceleration.
- Thereafter, the maximum demanded lateral acceleration falls off with increase of Φ_{V_T}

Further simulation studies are therefore carried out at an azimuth angle of 45°, at which angle the lateral acceleration demanded on the weapon is near its maximum value.

History of lateral acceleration demanded on weapon

Till now, the studies were directed towards assessing the influences of the guidance gain and azimuth angle Φ_{VT} on the *maximum value* of the demanded lateral acceleration. *The focus was only on a spot value of lateral acceleration.* A study is now undertaken to assess the *influence of the guidance gain and the* level of the longitudinal acceleration on the lateral acceleration demanded on the weapon *throughout its duration of flight.*

Lateral acceleration as a function of guidance gain

The lateral acceleration demanded on the weapon and the flight speed achieved by the weapon throughout its flight is tabulated in *Table C-1, as a function of the guidance gain.* The longitudinal acceleration is kept at a constant value of 200 m/s^2, throughout the simulation studies. The programme is run for Φ_{V_T} of 45°. The results are also plotted in Fig. C-2. The following observations are made:

(A) At all values of the guidance gain,

- During the initial part of its trajectory, the lateral acceleration demanded on the weapon *reaches a minimum value of less than* $10 m/s^2$.
- Thereafter, the lateral acceleration increases to very high values towards interception with the target.

- The time for the occurrence of this minimum lateral acceleration is however dependent on the value of the guidance gain and the speed of the weapon.
- *Increase in the value of the guidance gain, together with decrease in the speed of the weapon, lowers the time of occurrence of the point of minimum lateral acceleration.*

(B) At a guidance gain of 2.0,

- The lateral acceleration demanded on the weapon increases as it nears the target and demands its maximum value of 690.25 m/s^2 at its interception with the target.

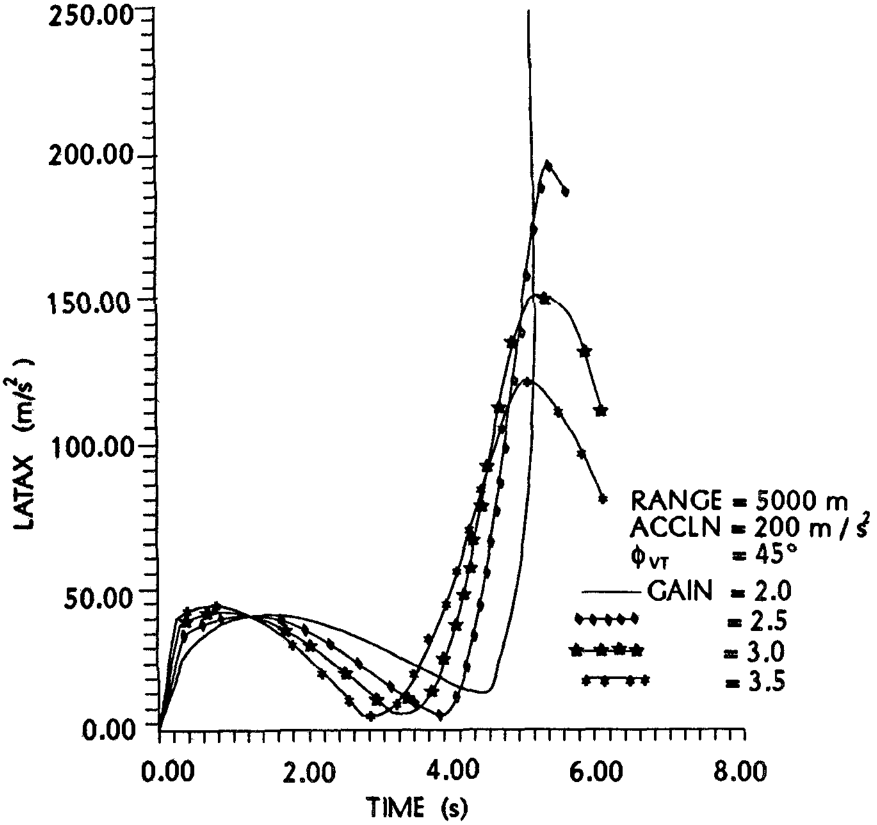


Figure C-2. Effect of guidance gain on latax history

Table C-1. Continuous boosting trajectory—History of lateral acceleration demanded on the weapon, as a function of guidance gain

	Demanded lateral acceleration and the speed achieved by the weapon at a guidance gain of:							
	2.0		*2.5*		*3.0*		*3.5*	
Time	*Latax*	*Speed*	*Latax*	*Speed*	*Latax*	*Speed*	*Latax*	*Speed*
0.0	0.0	34.77	0.0	34.77	0.0	34.77	0.0	34.77
0.2	23.11	74.90	28.70	75.00	34.21	75.11	39.64	75.25
0.4	26.13	115.32	32.10	115.68	37.85	116.11	43.38	116.61
0.6	28.84	155.99	34.92	156.74	40.57	157.63	45.80	158.64
0.8	31.21	196.90	37.10	198.13	42.28	199.57	46.80	201.18
1.0	33.19	238.01	38.57	239.81	42.92	241.86	46.34	244.08
1.2	34.75	279.31	39.28	281.72	42.44	284.37	44.38	287.19
1.4	35.85	320.78	39.19	323.80	40.81	326.99	40.94	330.22
1.6	36.45	362.39	38.27	365.97	38.01	369.61	36.05	373.12
1.8	36.53	404.10	36.48	408.17	34.05	412.09	29.81	415.68
2.0	36.05	445.88	33.79	450.31	28.96	454.33	22.31	457.75
2.2	34.96	487.69	30.21	492.32	22.79	496.20	13.78	499.18
2.4	33.24	529.47	25.71	534.10	15.67	537.58	5.42	539.85
2.6	30.85	571.19	20.34	575.58	8.10	578.39	8.89	579.67
2.8	27.75	612.78	14.18	616.68	5.73	618.53	19.89	618.56
3.0	23.91	654.20	7.91	657.31	14.18	657.93	32.02	656.49
3.2	19.33	695.38	6.52	697.40	24.96	696.52	44.70	693.45
3.4	14.14	736.26	13.90	736.88	36.71	734.28	57.64	729.46
3.6	9.11	776.78	23.92	775.69	49.13	771.19	70.60	764.58
3.8	8.18	816.86	35.28	813.77	62.03	807.25	83.27	798.90
4.0	14.56	856.43	47.75	851.10	75.22	842.52	95.36	832.54
4.2	24.50	895.42	61.25	887.63	88.45	877.04	106.49	865.65
4.4	36.68	933.74	75.73	923.38	101.45	910.92	116.30	898.40
4.6	51.08	971.32	91.10	958.35	113.86	944.28	124.36	930.98
4.8	68.05	1008.06	107.29	992.57	125.27	977.26	130.19	963.59
5.0	88.21	1043.87	124.10	1026.13	135.12	1010.05	133.32	996.43
5.2	112.57	1078.64	141.31	1059.10	142.70	1042.84	133.23	1029.68
5.4	142.98	1112.26	158.50	1091.65	147.08	1075.83	129.40	1063.50
5.6	183.32	1144.54	174.87	1123.94	146.86	1109.24	121.34	1098.03
5.8	244.83	1175.18	188.58	1156.21	139.76	1143.33	108.68	1133.38
6.0	430.42	1202.97	191.58	1188.83	120.33	1178.35	91.41	1169.60
6.02190	690.25	1205.37						
6.04320			182.63	1196.01				
6.06630					108.07	1190.25		
6.08960							83.82	1186.13

(C) At guidance gain higher than 2.0,

- The lateral acceleration demanded on the weapon also increases as it nears the target. The lateral acceleration drops to a lower value at interception with the target.
- The maximum value of the lateral acceleration demanded at interception decreases with increase in the guidance gain.
- *Since at values of the guidance gain higher than 2.0, the maximum lateral acceleration demanded on the weapon, and the time for the occurrence of the initial minimum lateral acceleration decreases in magnitude, the use of guidance gain higher than 2.0 is selected in design.*

Further simulation studies are therefore conducted at a guidance gain of 2.5.

Lateral Acceleration Demanded on Weapon·as a Function of the Longitudinal Acceleration

The computer simulation programme is now run at a guidance *gain of 2.5.* The history of lateral acceleration demanded on the weapon is tabulated in *Table C-2 as a function of the longitudinal acceleration.* The speed achieved by the weapon is also tabulated as a

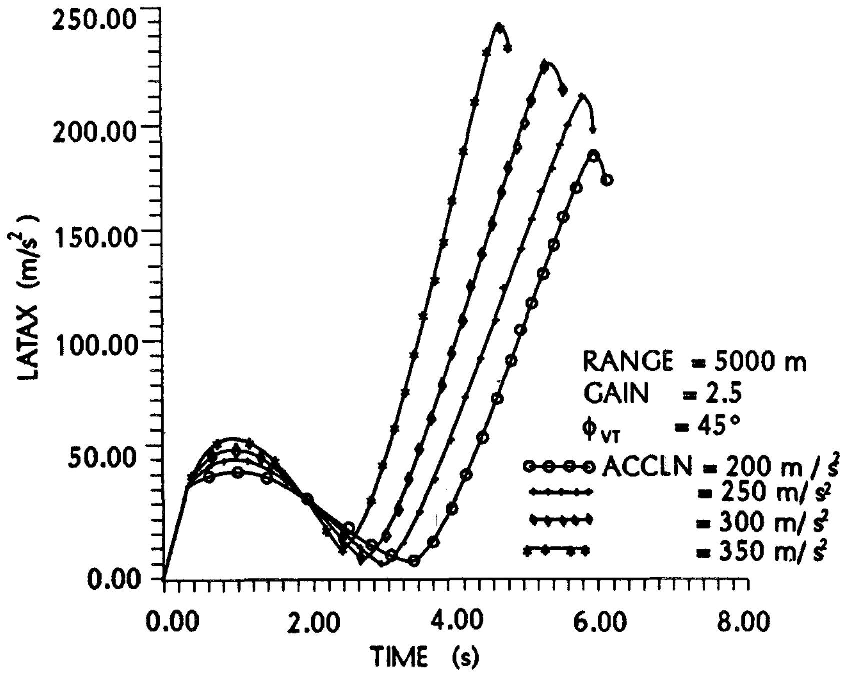


Figure C-3. Effect of longitudinal acceleration on latax history

function of the flight time. The results of these simulation studies are also plotted in *Fig. C-3.* The following observations are made:

Table C.2 Continuous boosting trajectory—History of lateral acceleration as a function of longitudinal acceleration

	Demanded lateral acceleration (m/s²) at a longitudinal acceleration of:							
Time	*350*		*300*		*250*		*200*	
	Latax	*Speed*	*Latax*	*Speed*	*Latax*	*Speed*	*Latax*	*Speed*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*
0.0	0.0	34.77	0.0	34.77	0.0	34.77	0.0	34.77
0.0	0.0	34.77	0.0	34.77	0.0	34.77	0.0	34.77
0.2	31.86	104.98	30.80	94.98	29.75	84.99	28.70	75.00
0.4	38.00	175.61	36.04	155.63	34.07	135.65	32.10	115.68
0.6	42.97	246.63	40.30	216.65	37.63	186.69	34.92	156.74
0.8	46.53	317.99	43.46	278.02	40.31	238.06	37.10	198.13
1.0	48.55	389.63	45.36	339.66	42.04	289.72	38.57	239.81
1.2	48.89	461.48	45.92	401.53	42.72	341.60	39.28	281.72
1.4	47.44	533.44	45.04	463.52	42.30	393.64	39.19	323.80
1.6	44.11	605.40	42.66	525.55	40.72	445.74	38.27	365.97
1.8	38.85	677.24	38.73	587.51	37.95	497.82	36.48	408.17
2.0	31.67	748.83	33.21	649.30	33.96	549.78	33.79	450.31
2.2	22.77	820.05	26.16	710.78	28.73	601.53	30.21	492.32
2.4	13.51	890.75	17.85	771.85	22.31	652.96	25.71	534.10
2.6	12.45	960.82	10.09	832.39	14.93	703.97	20.34	575.58
2.8	25.12	1030.1	12.37	892.30	8.19	754.48	14.18	616.68
3.0	42.88	1098.6	24.75	951.48	10.40	804.38	7.91	657.31
3.2	63.52	1166.2	40.51	1009.9	20.92	853.59	6.52	697.40
3.4	86.56	1232.8	58.47	1067.4	34.05	902.06	13.90	736.88
3.6	111.76	1298.5	78.33	1124.0	48.86	949.72	23.92	775.69
3.8	138.81	1363.2	99.91	1179.7	65.15	996.53	35.28	813.77
4.0	167.23	1427.2	123.02	1234.5	82.80	1042.5	47.75	851.10
4.2	196.14	1490.5	147.34	1288.5	101.70	1087.6	61.25	887.63
4..4	223.68	1553.5	172.34	1341.9	121.67	1131.9	75.73	923.38
4.6	244.09	1616.5	197.04	1394.8	142.47	1175.6	91.10	958.35
4.7130	237.00	1652.4						
4.8			219.12	1447.6	163.63	1218.6	107.28	992.57
5.0			229.09	1500.5	184.35	1261.3	124.10	1026.13
5.05010			220.04	1513.9				
5.2					202.83	1303.8	141.31	1059.10
5.4					212.57	1346.5	158.50	1091.65
5.47660					201.89	1363.1		
5.6							174.87	1123.94
5.8							188.58	1156.21
6.0							191.58	1186.83
6.04320							182.63	1196.01

(A) At all values of the longitudinal acceleration,

- In the initial phase of flight, the lateral acceleration demanded on the weapon *reaches a minimum value of less than 13* m/s^2.
- Thereafter, there is a sharp increase in the demanded lateral acceleration with increase in boosting time. It reaches a maximum value near the time of interception with the target.

(B) The time of occurrence of the initial minimum in the value of the lateral acceleration decreases with increase in the level of longitudinal acceleration and the consequent increase in the speed of the weapon.

- Higher the speed of the weapon, the earlier is the time of occurrence of the minimum lateral acceleration.
- However, the higher the speed, the higher also is the magnitude of the minimum lateral acceleration.

(C) In the period till the minimum lateral acceleration is reached,

- The lateral acceleration demanded on the weapon never exceeds $50m/s^2$.
- This value is lower, by an order of magnitude, compared to the lateral acceleration demanded on the weapon near its interception with the target.

Time for Occurrence of Minimum Lateral Acceleration

It is thus seen that the time of occurrence of the minimum lateral acceleration is strongly influenced both by the guidance gain and the speed achieved by the weapon under its longitudinal acceleration. While increase in the guidance gain decreases the time of occurrence of the minimum lateral acceleration, increase in the speed of the weapon increases the time of occurrence of the minimum lateral acceleration. This functional dependency is plotted as a carpet plot in Fig. C-4. In this carpet plot, instead of the speed of the weapon the longitudinal acceleration is used, since the speed of the weapon is a direct function of the acceleration. Based on the foregoing results, to achieve minimum lateral acceleration within 3.2 s in the initial phase of its flight, it is recommended that we limit further simulation studies to guidance gain greater than 2.5 and speed of the weapon greater than 500 m/s.

We have also seen that after the point of occurrence of the minimum lateral acceleration is reached, further increase in the speed of the weapon also increases the demands on the lateral acceleration on the weapon. At this stage of the studies, it is therefore

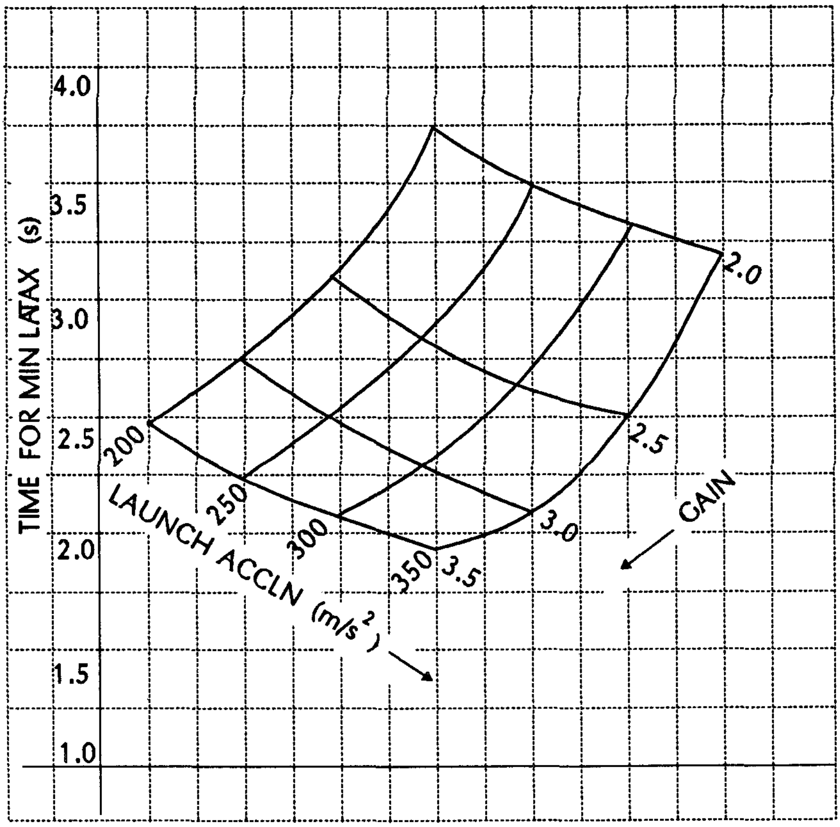


Figure C-4. Effect of gain and launch acceleration, on boosting time for minmum latax

advisable to check if there is any advantage in terminating the boost acceleration after the minimum lateral acceleration is reached. After all, the system does not know *apriori* that the thrusting would be terminated after the minimum lateral acceleration has been demanded!

If the designer chooses to terminate the boost acceleration at this point of time, then he has the choice of one of two further propulsion options to consider; either sustain the weapon at the boost-end speed till interception, or coast the weapon till interception with the target.

These propulsion options are listed earlier as Case 2 and Case 3, respectively under "thrusting programmes". Detailed studies on the implications of choosing either of these two propulsion options are studied now.

CASE 2. THE WEAPON IS IMPARTED A HIGH LEVEL OF BOOST ACCELERATION FOR A SHORT PERIOD OF TIME, FOLLOWED BY SUSTAINING IT AT THE BOOST-END SPEED, TILL *INTERCEPTION WITH THE TARGET.*

The simulation studies are now directed towards assessing the history of lateral acceleration on weapons that are boosted for a short duration of time, followed by sustaining it at the boost-end speed till interception with the target is achieved. For the purposes of this study, the boost acceleration is kept constant at 200 m/s^2. The boosting time is varied from 2.5 s to 3 s. The lateral acceleration demanded on the weapon till its interception with the target is presented in the Table C-3. The results are also plotted in Fig. C-5.

The following observations are made:

- In the initial boosting phase, the lateral acceleration demanded on the weapon does not exceed 40 m/s^2. This is a low level of lateral acceleration.
- During the sustained phase of flight the lateral acceleration demanded on the weapon keeps reducing as the weapon nears the target. This implies that during the sustaining phase of flight, the trajectory of the weapon has a tendency to become a *constant bearing course* as it nears the point of interception with the target. In all the cases considered, the final miss distance achieved is less than 1 m. These are advantages for efficient weapon design.
- It is also noticed that the higher the boost-end speed, the lower is the demands on the lateral acceleration. Boost-end speed of 657.11 m/s leads to the lateral acceleration of only 1.24 m/s^2 at interception. This is practically leading to a constant bearing course, when the lateral acceleration demanded on the weapon is zero.

It is therefore recommended that the weapon be boosted till a speed greater than 500 m/s is reached and then sustained at this speed.

Effect of command updating time and the rise time on the miss distance.

The effect of the command updating time and the rise time, on the miss distance achieved by the weapon is tabulated in Table C-4.

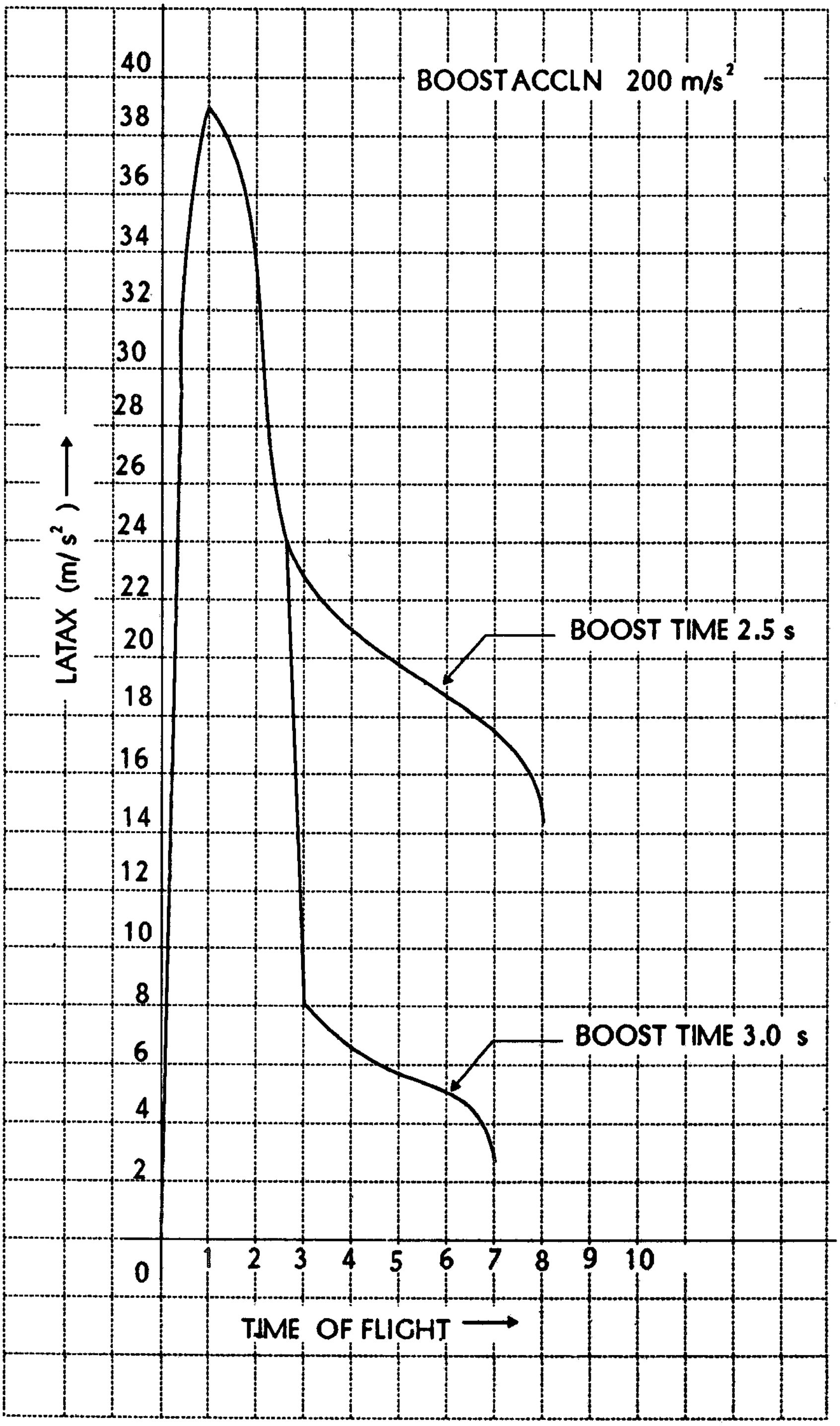


Figure C-5. History of lateral acceleration in boost-sustained trajectory

Table C-3. Boost-sustained trajectory—History of lateral acceleration demanded on the weapon in m/s² as a function of boost acceleration and boost cut off time

Boost acceleration = 200 m/s², Guidance gain = 2.5

Weapon sustained at its boost-end speed till interception with target

Time of flight	*Lateral acceleration*	*Speed of flight*	*Lateral acceleration*	*Speed of flight*
0.0	0.0	34.77	0.0	34.77
0.2	28.70	75.0	28.70	75.0
0.4	32.10	115.68	32.10	115.68
0.6	34.92	156.74	34.92	156.74
0.8	37.10	198.13	37.10	198.13
1.0	38.57	239.81	38.57	239.81
1.2	39.28	281.72	39.28	281.72
1.4	39.19	323.80	39.19	323.80
1.6	38.27	365.97	38.27	365.97
1.8	36.48	408.17	36.48	408.17
2.0	33.79	450.31	33.79	450.31
2.2	30.21	492.32	30.21	492.32
2.4	25.71	534.10	25.71	534.10
2.6	23.03	554.68	20.34	575.58
2.8	22.77	554.68	14.18	616.68
3.0	22.51	554.68	7.91	657.11
3.2	22.25	554.68	7.79	657.11
3.4	22.00	554.68	7.64	657.11
3.6	21.75	554.68	7.49	657.11
3.8	21.50	554.68	7.34	657.11
4.0	21.25	554.68	7.18	657.11
4.2	21.01	554.68	7.02	657.11
4.4	20.77	554.68	6.86	657.11
4.6	20.53	554.68	6.68	657.11
4.8	20.29	554.68	6.51	657.11
5.0	20.05	554.68	6.32	657.11
5.2	19.81	554.68	6.13	657.11
5.4	19.57	554.68	5.92	657.11
5.6	19.33	554.68	5.71	657.11
5.8	19.10	554.68	5.48	657.11
6.0	18.86	554.68	5.23	657.11
6.2	18.62	554.68	4.96	657.11
6.4	18.37	554.68	4.65	657.11
6.6	18.12	554.68	4.29	657.11
6.8	17.87	554.68	3.83	657.11
7.0	17.60	554.68	3.14	657.11
7.14120			1.24	657.11
7.2	17.31	554.68		
7.4	16.98	554.68		
7.6	16.56	554.68		
7.8	15.70	554.68		
7.84590	14.33	554.68		

It is seen from the table that miss distance less than 0.1 m can be achieved, if:

Table C-4. Effect of command updating time and the rise time on the miss distance

	Miss distance (m), achieved in a response time of:									
t_{rise} →	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
t_{update} ↓										
0.01	*0.03*	0.00	*0.01*	*0.01*	*0.05*	*0.01*	*0.05*	*0.03*	*0.05*	*0.05*
0.02	*0.05*	*0.04*	*0.01*	*0.01*	*0.05*	*0.03*	*0.02*	*0.05*	*0.04*	*0.02*
0.03	*0.00*	*0.03*	*0.06*	*0.06*	*0.06*	*0.06*	*0.02*	*0.02*	*0.07*	*0.03*
0.04	*0.07*	*0.00*	*0.03*	*0.05*	*0.07*	*0.04*	*0.05*	*0.03*	*0.06*	*0.02*
0.05	*0.03*	*0.04*	*0.01*	*0.05*	*0.03*	*0.05*	*0.04*	*0.01*	*004*	*0.06*
0.06	*0.07*	*0.01*	*0.01*	*0.01*	*0.04*	*0.04*	*0.01*	*0.04*	*0.00*	*0.03*
0.07	*0.05*	*0.00*	*0.03*	*0.01*	*0.02*	*0.07*	*0.04*	*0.07*	*0.03*	*0.07*
0.08	*0.04*	*0.03*	*0.03*	*0.05*	*0.05*	*0.02*	*0.06*	*0.02*	*0.06*	*0.04*
0.09	*0.07*	*0.02*	*0.07*	*0.01*	*0.06*	*0.04*	*0.01*	*0.05*	*0.02*	*0.01*
0.10	*0.06*	*0.06*	*0.02*	*0.02*	*0.01*	*0.05*	*0.02*	*0.00*	*0.07*	*0.00*
0.10	*0.06*	*0.06*	*0.02*	*0.02*	*0.01*	*0.05*	*0.02*	*0.00*	*0.07*	*0.00*
0.20	0.05	0.01	0.04	0.03	0.04	0.07	0.06	0.07	0.05	0.05
0.30	0.03	0.07	0.07	0.04	0.05	0.04	0.08	0.10	0.12	0.16
0.40	0.02	0.08	0.08	0.12	0.17	0.20	0.24	0.29	0.32	0.34
0.50	0.06	0.08	0.04	0.07	0.11	0.16	0.22	0.29	0.36	0.44
0.60	0.12	0.22	0.29	0.36	0.42	0.45	0.47	0.48	0.48	0.46
0.70	0.40	0.50	0.61	0.72	0.83	0.94	1.06	1.18	1.28	1.36
0.80	0.35	0.44	0.50	0.54	0.56	0.57	0.55	0.52	0.47	0.40
0.90	0.52	0.59	0.64	0.66	0.67	0.65	0.61	0.55	0.46	0.37
1.00	0.56	0.80	1.01	1.20	1.37	1.51	1.62	1.71	1.78	1.82

- The value of t_{rise} does not exceed 0.10 s and
- The updating interval does not exceed t_{update} 0.2 s.

Table C-5. Effect of command updating time and the rise time on the maximum lateral acceleration demanded on the weapon

	Lateral acceleration, in m/s^2, demanded on the weapon for a response time of:									
t_{rise} → t_{update} ↓	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*
0/01	39.92	40.41	40.88	41.34	41.78	42.21	42.63	43.04	43.45	43.85
0.02	40.40	40.89	41.35	41.81	42.25	42.68	43.10	43.51	43.92	44.32
0.03	40.87	41.35	41.82	42.27	42.71	43.14	43.56	43.97	44.38	44.78
0.04	41.33	41.81	42.28	42.73	43.17	43.60	44.02	44.43	44.84	45.24
0.05	41.79	42.27	42.73	43.18	43.62	44.05	44.48	44.89	45.30	45.70
0.06	42.24	42.72	43.18	44.91	44.07	44.50	44.93	330.0	45.75	46.15
0.07	42.69	43.17	43.63	44.08	44.52	44.95	45.38	45.79	46.20	46.61
0.08	69.58	62.53	44.08	65.06	256.0	693.36	45.82	46.24	46.66	47.06
0.09	43.57	44.06	44.52	44.96	45.41	45.84	46.26	46.68	47.10	47.51
0.10	44.02	44.48	44.95	45.41	45.85	46.26	46.71	47.13	47.54	47.94
0.20	48.29	48.77	49.23	49.67	50.1	50.55	51.03	51.50	51.96	52.39
0.30	52.48	52.99	53.48	53.97	54.44	54.90	55.34	55.77	56.19	56.59
0.40	65.00	76.53	84.91	90.74	94.93	98.00	100.41	102.34	100.68	95.90
0.50	61.02	61.55	62.07	62.59	63.09	63.59	64.07	64.55	65.02	65.47
0.60	65.09	65.79	66.48	67.16	67.83	68.50	69.16	69.81	70.45	71.08
0.70	69.67	70.21	70.74	14744	12988	8570.5	6191.0	4842.5	3898.1	3172.4
0.80	75.32	76.00	76.67	77.34	77.99	78.64	79.28	79.91	80.53	81.15
0.90	79.04	79.79	80.53	81.27	82.00	82.73	83.44	84.15	84.86	85.55
1.00	81.16	81.94	82.72	83.49	84.25	85.00	85.75	86.50	87.24	87.97

Effect of command updating time and the rise time on the maximum lateral acceleration demanded on the weapon

The effect of command updating time and the rise time on the maximum lateral acceleration demanded on the weapon is presented in Table C-5.

It is seen from the table that the maximum lateral acceleration less than 60 m/s^2 can be achieved, if:

(a) The value of t_{rise} does not exceed 0.10 s, and

(b) The updating interval does not exceed 0.30 s.

(c) *However, to restrict the miss distance to 0.10 m, it is recommended that*

- *the value of rise time does not exceed 0.10 s and*
- *the updating interval does not exceed 0.2 s.*

CASE 3. THE WEAPON IS ACCELERATED TO A VERY HIGH VELOCITY IN A SHORT DURATION BOOSTING PHASE. THE WEAPON IS THEN COASTED TILL AN INTERCEPTION WITH THE TARGET IS ACHIEVED.

The simulation studies are now directed towards assessing the history of lateral acceleration on weapons that are boosted for a short duration, followed by a coasting phase of flight. In the coasting phase, the boost-end momentum of the weapon is expended in overcoming the aerodynamic drag force of the weapon. The simulation studies are conducted keeping the booster acceleration at a value of 200 m/s^2 and boosting the weapon for a duration of 2.5 s. The weapon is then coasted till interception with the target is achieved.

The aerodynamic drag deceleration during the coasting phase is given by:

$$Drag\ deceleration = \frac{V^2 * C_d}{\mu_{be}} \tag{5.17}$$

where,

V is the flight speed,

C_d is the coefficient of drag of the weapon (This is a function of the flight speed),

μ is the factor defined by $\dfrac{2 * m_{be}}{\rho * S}$,

m_{be} is the boost-end mass of the weapon,

S is the reference area of the weapon configuration, and

ρ is the density of air relevant to the altitude of flight.

It is noticed that the drag deceleration is directly proportional to the drag coefficient and inversely proportional to the boost-end value of the factor μ. For the purposes of this simulation studies, C_d value of 0.3 has been used, which is the value typical for tactical missiles in the supersonic range. The influence of the factor μ_{be} on the lateral acceleration demanded during the coasting period of flight is presented in Table C-6. The results are also plotted in Fig. C-6. The following observations are made:

- In the coasting phase, the lateral acceleration demanded on the weapon and the time of flight till interception with the target increases as the value of μ decreases. This is indicative of a higher drag deceleration associated with lower values of μ.

- For values of μ higher than 6000 however, the lateral acceleration demanded on the weapon is either a constant or keeps reducing very gradually. The value of the lateral acceleration however does not exceed 24 m/s^2 in the coasting phase.

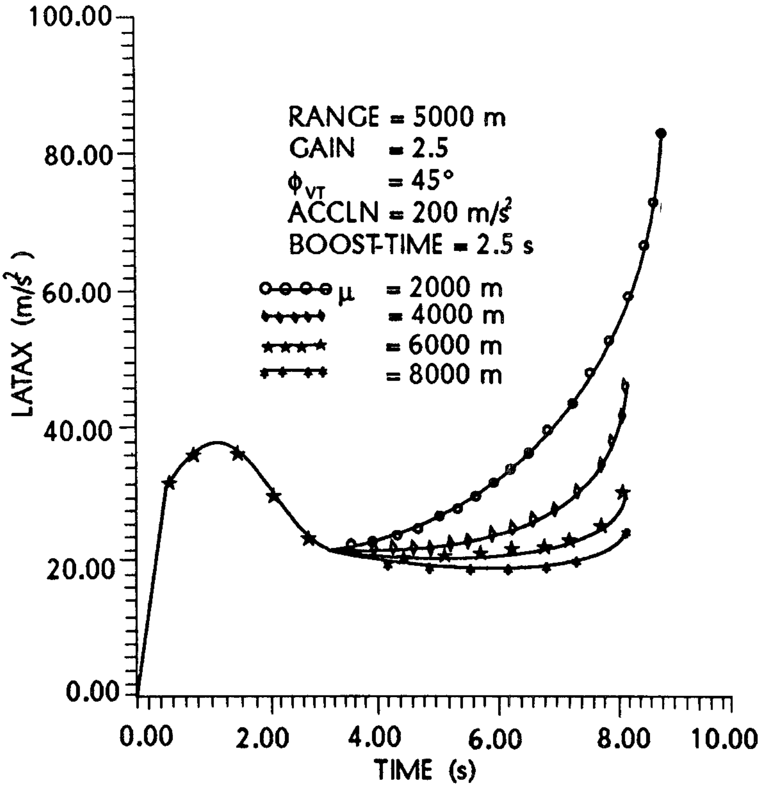


Figure C-6. Effect of μ on latax history

Table C-6. Boost-coast trajectory-History of demanded lateral acceleration and flight speed, as a function of μ_{be}

Time of flight	*Demanded lateral acceleration (m/s^2) and flight speed (m/s) as a function of μ_{be}*							
	$\mu_{be} = 2000$		$\mu_{be} = 4000$		$\mu_{be} = 6000$		$\mu_{be} = 8000$	
	Latax	*Speed*	*Latax*	*speed*	*Latax*	*Speed*	*Latax*	*Speed*
.2	28.70	75.00	28.70	75.00	28.70	75.00	28.70	75.00
0.4	32.10	115.68	32.10	115.68	32.10	115.68	32.10	115.68
0.6	34.92	156.74	34.92	156.74	34.92	156.74	34.92	156.74
0.8	37.10	198.13	37.10	198.13	37.10	198.13	37.10	198.13
1.0	38.57	239.81	38.57	239.81	38.57	239.81	38.57	239.81
1.2	39.28	281.72	39.28	281.72	39.28	281.72	39.28	281.72
1.4	39.19	323.80	39.19	323.80	39.19	323.80	39.19	323.80
1.6	38.27	365.97	38.27	365.97	38.27	365.97	38.27	365.97
1.8	36.48	408.17	36.48	408.17	36.48	408.17	36.48	408.17
2.0	33.79	450.31	33.79	450.31	33.79	450.31	33.79	450.31
2.2	30.21	492.32	30.21	492.32	30.21	492.32	30.21	492.32
2.4	25.71	534.10	25.71	534.10	25.71	534.10	25.71	534.10
2.6	23.48	551.05	23.20	553.33	23.10	554.09	23.05	554.48
2.8	24.17	543.62	23.33	550.28	23.04	552.54	22.90	553.67
3.0	24.86	536.51	23.48	547.32	23.00	551.02	22.75	552.89
3.2	25.55	529.69	23.64	544.44	22.96	549.54	22.61	552.13
3.4	26.25	523.17	23.81	541.65	22.93	548.10	22.47	551.38
3.6	26.94	516.93	24.00	538.94	22.91	546.70	22.34	550.66
3.8	27.65	510.97	24.20	536.31	22.90	545.33	22.21	549.96
4.0	28.36	505.27	24.41	533.77	22.90	544.00	22.10	549.27
4.2	29.08	499.84	24.65	531.31	22.91	542.71	21.99	548.61
4.4	29.82	494.66	24.90	528.93	22.93	541.47	21.88	547.96
4.6	30.58	489.73	25.17	526.64	22.97	540.26	21.79	547.34
4.8	31.36	485.04	25.46	524.44	23.02	539.08	21.70	546.73
5.0	32.16	480.60	25.77	522.32	23.09	537.95	21.63	546.14
5.2	32.99	476.40	26.11	520.29	23.18	536.87	21.56	545.57
5.4	33.86	472.43	26.48	518.35	23.28	535.82	21.50	545.02
5.6	34.77	468.71	26.89	516.51	23.40	534.82	21.46	544.50
5.8	35.73	465.22	27.33	514.75	23.55	533.86	21.43	543.99
6.0	36.74	461.36	27.81	513.09	23.72	532.95	21.42	543.50
6.2	37.82	458.96	28.34	511.54	23.93	532.09	21.42	543.04
6.4	38.97	456.19	28.93	510.09	23.17	531.28	21.44	542.60
6.6	40.22	453.68	29.58	508.74	24.46	530.52	21.49	542.19
6.8	41.59	541.43	30.33	507.52	24.79	529.82	21.57	541.81
7.0	43.09	449.45	31.18	506.42	25.20	529.19	21.69	541.45
7.2	44.78	447.75	32.17	505.47	25.70	528.63	21.86	541.13
7.4	46.68	446.36	33.36	504.66	26.33	528.15	22.11	540.85
7.6	48.88	445.30	34.84	504.04	27.18	527.77	22.49	540.62
7.8	51.49	444.61	36.80	503.63	28.44	527.51	23.17	540.46
7.92020							24.50	540.41
7.99410					31.93	527.46		
8.0	54.71	444.34	39.83	503.50				
8.14420			46.06	503.72				
8.2	58.91	444.59						
8.4	65.11	445.51						
8.6	79.82	447.55						
8.6135	84.19	447.78						

CONCLUSION

Considering the results of the computer simulation studies, the set of compatible essential system functional features needed for the design of the shoulder fired weapon system can be specified as below:

Case 1.

The guidance gain	≥ 2.5
The rise time of the weapon in its short period mode	0.10 s
Command updating interval not to exceed	0.20 s
The frequency at which the commands need to be updated.	≤ 0.1 s
The longitudinal acceleration needed for launching the weapon,	$\geq 200\ m/s^2$

$2.5\ s \leq$ *Boosting time* $\leq 3.0\ s$.

$$\frac{V_M}{V_T} \geq 2.0$$

Case 2. Boost-sustained policy

Longitudinal acceleration in the sustained phase of flight	0.0
Maintain boost-end speed ratio $\frac{V_M}{V_T}$	≥ 2.0 till interception
Maximum lateral acceleration demanded on the weapon	$60.00\ m/s^2$

Or

Case 3. Boost-coast policy

The longitudinal acceleration needed for launching the weapon	$\geq 200\ m/s^2$

$2.5\ s. \leq$ *Boosting time* $\leq 3.0\ s$

Coast the weapon till interception

Boost-end value, μ	≥ 6000
The maximum lateral acceleration demanded on the weapon	$39.28\ m/s^2$
Miss distance	$\leq 0.1\ m$

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1. A system study on shoulder fired missiles. R. Balakrishnan and K. Anandha Narayanan. DRDL Technical Report, 1993.

Appendix D

CONFIGURATION SOLUTIONS

The method developed in Chapter 6 is applied to identify the solution regions of a tail-controlled configuration and a canard-controlled configuration. The wing-controlled configurations, however, can be treated as a special case of a canard-controlled configurations.

SOLUTIONS TO TAIL-CONTROLLED CONFIGURATIONS

The relevant normalised equations that are required to be solved for arriving at the normalised configuration solutions for tail-controlled configurations are enumerated below:

$$\bar{a}_n + \bar{a}_w + \bar{a}_t = 1 \tag{6.14}$$

$$\begin{pmatrix} \bar{a}_n \geq 0.0 \\ \bar{a}_w \geq 0.0 \\ \bar{a}_t \geq 0.0 \end{pmatrix} \tag{6.14a}$$

$$(\bar{a}_n * \lambda_n) + (\bar{a}_w * \lambda_w) + (\bar{a}_t * \lambda_t) = 1 \tag{6.15}$$

$$\left| (\bar{a}_c * \lambda_c) \right| < 1 \tag{6.17}$$

$$\left(\bar{a}_c = \bar{a}_t * \frac{k_{b(t)} + k_{t(b)}}{K_{b(t)} + K_{t(b)}} \right), \textit{in the case of tail control} \tag{6.20}$$

$$2.0 \leq \left(\frac{s}{r}\right) \leq 3.5 \tag{6.21}$$

$$0.645 \leq \left(\frac{k_{b(t)} + k_{t(b)}}{K_{b(t)} + K_{t(b)}} \right) \leq 0.75 \tag{6.22}$$

$$4.0 < \left| \lambda_c \right| < 8.0 \tag{6.23}$$

$0.0 < \bar{a}_n < 0.3$ (6.26a), in the case of tapered, lifting forebodies, and

$\bar{a}_n = 0.0$ (6.26b), in the case of blunt, non-lifting forebodies.

$$- 8.0 < \lambda_n < -3.0 \qquad (6.27)$$

In efficient tail-controlled configuration designs, the control moment should satisfy the inequality constraint given by Eqn (6.17). Further, since the tail control arm should also satisfy the inequality constraint given by Eqn (6.23), the specification for the tail control force is an inequality given by:

$$0.125 < \bar{a}_c < 0.25 \qquad (T.1)$$

For the above range in the values of $\bar{a}_c$, depending upon the choice of the $\frac{s}{r}$ ratio for the tail control surfaces, along with their relevant range in the values of the lift interference factors, the range in the value of $\bar{a}_t$ can be calculated. Thus,

$$0.1666 < \bar{a}_t < 0.3876 \qquad (T.2)$$

Using this permissible spread in the values of $\bar{a}_t$ in Eqn (6.14), the spread in the values of $(\bar{a}_n + \bar{a}_w)$ is obtained as:

$$0.6124 < (\bar{a}_n + \bar{a}_w) < 0.8334 \qquad (T.3)$$

Now, let us consider the moments contributed by the three lifting elements. The control moment should satisfy the inequality given by Eqn (6.17). Therefore, depending upon the choice of the $\frac{s}{r}$ ratio of the control surfaces, the range in the moment caused by the tail lift can be obtained as given in relation (T.4) below. It is *assumed* that $\lambda_c \equiv \lambda_t$

$$1.333 < \bar{a}_t * \lambda_t < 1.550 \qquad (T.4)$$

Using this range in the tail moment in Eqn (6.15), the spread in the combined moments contributed by the nose lift and the wing lift is got as

$$-0.55 \le \left(\bar{a}_n * \lambda_n + \bar{a}_w * \lambda_w\right) \le -0.333 \qquad \text{(T.5)}$$

Now two cases arise, depending upon whether the forebody of the configuration is a lifting forebody or a non-lifting forebody.

Case 1. Configuration with a lifting forebody

The permissible spread in the values of the nose lift is given in Eqn *(6.26a)* as

$$0.0 < \bar{a}_n < 0.3 \qquad \text{(6.26a)}$$

Accounting for this spread in $\bar{a}_n$ in Eqn (T.3), the range in the value of $\bar{a}_w$ is obtained as

$$0.3124 < \bar{a}_w < 0.8334 \qquad \text{(T. 6a)}$$

Now let us consider the moment contributed by the nose lift. The range in the nose lift is given by Eqn (6.26a) and the range in its moment arm is given by Eqn (6.27). Therefore, the permissible spread in the nose moment is obtained as

$$-2.4 < \bar{a}_n * \lambda_n < 0.0 \qquad \text{(T.7)}$$

Accounting for this spread in the value of the nose moment in Eqn (T.5), the range in the moment contributed by the wing is obtained as

$$-0.55 < \bar{a}_w * \lambda_w < 2.07 \qquad \text{(T.8a)}$$

The permissible range in the value of the wing lift is given by (T.6a). Using this range in the moment Eqn (T.8a), the permissible range in the wing moment arm is given by Eqn (T.9a) below:

$$-1.76 < \lambda_w < 6.530 \qquad \text{(T.9a)}$$

Case 2. Configuration with non-lifting forebody

The value of the nose lift is given in Eqn (6.26b) as

$$\bar{a}_n = 0.0 \qquad \text{(6.26b)}$$

Accounting for $\bar{a}_n$ in Eqn (T.3), the range in the value of $\bar{a}_w$ is obtained as

$$0.6124 < \left(\bar{a}_W\right) < 0.8334 \qquad \text{(T.6b)}$$

Since the nose is not contributing any lift, the moment caused by it is zero. Using this fact in Eqn (T.5), the permissible range in the moment caused by the wing is got as

$$-0.55 < \left(\bar{a}_W * \lambda_W\right) < -0.33 \qquad \text{(T.8b)}$$

The permissible range in the value of the wing lift is given in (T.6b). Using this range in the value of wing lift in the moment Eqn (T.8b), the permissible range in the wing moment arm is given by the relation

$$-0.898 < \lambda_W < -0.396 \qquad \text{(T.9a)}$$

Thus, the normalised solutions for the family of tail control tactical weapons to meet the two systems level requirements on the configuration have been identified. These are summarised in Table D-1. The estimates made on the RAPIER configuration is also included in the table, by way of comparison.

Table D-1. The solution set for tail-controlled configurations

Entity	*Normalised solution range*	*Estimated value on RAPIER*
Tail control arm, λ_c	$4.0 < \lambda_c < 8.0$	5.603
Tail control lift-curve slope, $\bar{a}_c$	$0.125 < \bar{a}_c < 0.25$	0.179
Span to diameter ratio of tail controls	2.0< (s/r) controls < 3.5	2.86
Normalised tail lift curve slope, $\bar{a}_t$	$0.1666 < \bar{a}_t < 0.3876$	0.254
Normalised tail moment, $\left(\bar{a}_t * \lambda_t\right)$	$1.33 < \left(\bar{a}_t * \lambda_t\right) < 1.55$	1.423
Case 1: Normalised nose lift curve slope (configuration with tapered forebody), $\bar{a}_n$	$0.0 < \bar{a}_n < 0.3$	0.126

Contd..

Entity	*Normalised solution range*	*Estimated value on RAPIER*
Normalised moment arm for nose lift, λ_n	-8.0 < ln < -3.0	-6.235
Normalised moment of nose lift, $(\bar{a}_n * \lambda_n)$	$-2.4 < (\bar{a}_n * \lambda_n) < 0.0$	-0.786
Normalised wing lift, $\bar{a}_W$	$0.3124 < \bar{a}_W < 0.8334$	0.62
Normalised wing moment, $(\bar{a}_W * \lambda_W)$	$-0.55 < (\bar{a}_W * \lambda_W) < 2.07$	0.363
Normalised moment arm for wing lift, λ_W	$-1.76 < \lambda_W < 6.53$	0..585
Case 2: Normalised nose lift-curve slope (configuration with blunt forebody), a_n =	0.0	
Normalised wing lift curve slope, $\bar{a}_W$	$0.6124 < \bar{a}_W < 0.8334$	
Normalised wing moment, $(\bar{a}_W * \lambda_W)$	$-0.55 < (\bar{a}_W * \lambda_W) < -0.33$	
Normalised moment arm, for wing lift, λ_W	$-0.898 < \lambda_W < -0.395$	

SOLUTION SET FOR CANARD-CONTROLLED CONFIGURATIONS

The relevant normalised equations that are required to be solved for arriving at the normalised configuration solutions for canard-controlled configurations are enumerated below:

$$\bar{a}_n + \bar{a}_W + \bar{a}_t = 1 \tag{6.14}$$

$$\begin{pmatrix} \bar{a}_n \geq 0.0 \\ \bar{a}_W \geq 0.0 \\ \bar{a}_t \geq 0.0 \end{pmatrix} \tag{6.14a}$$

$$(\bar{a}_n * \lambda_n) + (\bar{a}_W * \lambda_W) + (\bar{a}_t * \lambda_t) = 1 \tag{6.15}$$

$$\left(\bar{a}_c = \bar{a}_{ca} * \frac{k_{b(ca)} + k_{ca(b)}}{K_{b(ca)} + K_{ca(b)}}\right), \text{ in the case of canard control} \tag{6.20}$$

$$2.0, \leq \left(\frac{s}{r}\right) \leq 3.5 \tag{6.21}$$

$$0.645 \leq \left(\frac{k_{b(ca)} + k_{ca(b)}}{K_{b(ca)} + K_{ca(b)}}\right) \leq 0.75 \tag{6.22}$$

$$-8.0 < \lambda_c < -4.0. \tag{6.23}$$

$$\left(\frac{Stall\lim it}{\alpha_{conf}}\right) \geq \left(1 + \frac{1}{(\bar{a}_c * \lambda_c)}\right) \tag{6.25}$$

$0.0 < \bar{a}_n <) \; 0.3$ (6.26a), in the case of tapered, lifting forebodies, and

$\bar{a}_n = 0.0$ (6.26b) in the case of blunt, non-lifting forebodies.

$$-8.0 < \lambda_n < -3.0 \tag{6.27}$$

In good canard configuration designs, the criterion in choosing the maximum angle of attack of the configuration is the avoidance of stalling of the canard control surfaces. If the canard control surfaces are of low aspect ratio, then stalling may not occur on them. However, at these high angles of deflection, the canard lift would be highly non-linear. In such situations, the ease of handling this non-linear behaviour in the control systems design would set the limit for the maximum angle of deflection of the canard controls. A practical limit for the maximum deflection of low aspect ratio canard control surfaces is around 25°.

Similarly, the flow around the cylindrical bodies of typical guided weapon configurations would separate, as the angle of attack of the configuration increases. It is therefore a general practice to limit maximum angle of attack on such configurations to about 10°.

In canard-control configurations, considering these two practical limits, one can work for:

$$\frac{stall\,limit}{\alpha_{conf}} \geq 2.5 \qquad \text{(Ca.1)}$$

One arrives at the range in the value of $|(\bar{a}_C * \lambda_C)|$, by solving the inequalities given by (Ca.1) along with Eqn (6.25). Thus,

$$|(\bar{a}_C * \lambda_C)| \leq 0.666 \qquad \text{(Ca.2)}$$

Considering the permissible range in the value of λ_C defined by Eqn (6.23), the range in the value of $\bar{a}_C$ is obtained as

$$0.0833 < \bar{a}_C < 0.1667 \qquad \text{(Ca.3)}$$

Depending upon the choice of the $\frac{s}{r}$ ratio for the canard control surfaces, the spread in the value of the canard lift can be determined. The relevant range in the values of the lift interference factors is used in such determination. Thus,

$$0.111 < \bar{a}_{Ca} < 0.2584 \qquad \text{(Ca.4)}$$

Using this range in the value of $\bar{a}_{Ca}$ in Eqn (6.14), the spread in the values of $(\bar{a}_n + \bar{a}_w)$ is obtained as:

$$0.7416 < (\bar{a}_n + \bar{a}_w) < 0.8889 \qquad \text{(Ca.5)}$$

The moments contributed by the three lifting elements are as follows. The control moment should satisfy the inequality given by (Ca.2). Therefore, depending upon the choice of the $\frac{s}{r}$ ratio of the control surfaces, the range in the moment caused by the canard lift is obtained as

$$-1.0336 < (\bar{a}_{Ca} * \lambda_{Ca}) < -0.8888 \qquad \text{(Ca.6)}$$

Using this range in the moment contributed by the canards in Eqn (6.15), the spread in the combined moments contributed by the nose lift and the wing lift is got as

$$1.888 < \left(\bar{a}_n * \lambda_n + \bar{a}_w * \lambda_w\right) < 2.0336 \qquad \text{(Ca.7)}$$

At this stage in the design two cases arise, depending upon whether the forebody of the configuration is a lifting forebody or a non-lifting forebody.

Case 1. Configuration with a lifting forebody

The permissible spread in the values of the nose lift is given in Eqn (6.26a) as

$$0.0 < \bar{a}_n < 0.3 \qquad \text{(6.26a)}$$

Accounting for this spread in $\bar{a}_n$ in the Eqn (Ca.5), the range in the value of $\bar{a}_w$ is obtained as given in the Eqn (Ca.8a) below:

$$0.4436 < \bar{a}_w < 0.8889 \qquad \text{(Ca.8a)}$$

Now let us consider the moment contributed by the nose lift. The range in the nose lift is given by Eqn (6.26a) and the range in the moment arm is given by Eqn (6.27). Therefore, the permissible spread in the nose moment is given by the relation

$$-2.40 < \bar{a}_n * \lambda_n < 0.0 \qquad \text{(Ca.9a)}$$

Accounting for this spread in the value of the nose moment in Eqn (Ca.7), the range in the moment contributed by the wing is obtained as

$$1.888 < \bar{a}_w * \lambda_w < 4.4336 \qquad \text{(Ca.10a)}$$

The permissible range in the value of the wing lift is given in (Ca.8a). Using this range in the moment Eqn (Ca.10a), the permissible range in the wing moment arm is given by the relation

$$2.125 < \lambda_w < 9.995 \qquad \text{(Ca.11a)}$$

Case 2. Configuration with non-lifting forebody

The value of the nose lift is given as

$$\bar{a}_n = 0.0 \qquad \text{(6.26b)}$$

Accounting for this value of $\bar{a}_n$ in Eqn (Ca.5), the range in the value of $\bar{a}_w$ is obtained as

$$0.7436 < \left(\bar{a}_W\right) < 0.889 \qquad \text{(Ca.8b)}$$

The moment contributed by the nose lift is zero. Using this fact in Eqn (Ca.7), the permissible range in the moment caused by the wing is got as

$$1.888 < \left(\bar{a}_W * \lambda_W\right) < 2.0336 \qquad \text{(Ca.10b)}$$

The permissible range in the value of the wing lift is given in (Ca.8b). Using this range in the value of wing lift in the moment Eqn (Ca.10b), the permissible range in the wing moment arm is obtained as

$$2.1247 < \lambda_W < 2.735 \qquad \text{(Ca.11b)}$$

Thus, the normalised solutions for the family of canard-control weapon to meet the two systems level requirements on the configuration have been identified. These are summarised in *Table D-2.* The estimated values of the corresponding parameters pertinent to the CROTALE and the TRISHUL configurations are also listed in the table for comparison. The shape of the canard in the CROTALE configuration is uncommon. The 'equivalent area' method given in the EDSU data sheets have therefore been adopted in the estimation of the effectiveness of the canards. The value of $\bar{a}_{Ca}$, thus estimated, falls slightly outside the acceptable range. This may be due to the inadequacy of the method adopted for its estimation.

Table D-2. The solution set for canard-controlled configurations

Entity	*Range in the values*	*Estimates on CROTALE*	*Estimates on TRISHUL*
Normalised control lift-curve slope, $\bar{a}_c$	$0.083 < \bar{a}_c < 0.166$	0.199 (derived after estimation of $\bar{a}_{Ca}$)	0.135
Normalised control moment arm, λ_c	$-8.0 < \lambda_c < -4.0$	-3.3436	-6.367
Span-to-diameter ratio of tail controls	2.0 > (s/r) >3.5	3.22	2.0
Normalised canard lift curve slope, $\bar{a}_{Ca}$	$0.111 < \bar{a}_{Ca}$ < 0.2584	0.299(estimates based on equivalent area method)	0.18
Normalised moment of canard lift $(\bar{a}_{Ca} * \lambda_{Ca})$	$-0.888 < (\bar{a}_{Ca} * \lambda_{Ca}) < -1.0336$	-1.0014	-1.146
Case 1. Lifting forebody			
nose lift curve slope, $\bar{a}_n$	$0.0 < \bar{a}_n < 0.3$	0.0616	0.224
Normalised moment arm for nose lift, λ_N	$-8.0 \leq \lambda_N \leq -3$	3.2677	-6.869
Normalised slope of wing lift, $\bar{a}_W$	$0.4436 < \bar{a}_W < 0.889$	0.6388	0.596
Normalised moment of nose lift, $(\bar{a}_n * \lambda_n)$	$-2.4 < (\bar{a}_n * \lambda_n) < 0.0$	0.2014	-1.539
Normalised moment of wing lift, $(\bar{a}_W * \lambda_W)$	$1.888 < (\bar{a}_W * \lambda_W) < 4.4336$	2.2028	3.685
Normalised moment arm for wing lift, λ_W	$2.125 < \lambda_W < 9.995$	3.4483	6.183
Case 2. Non-lifting forebody			
$\bar{a}_n$	$\bar{a}_n = 0.0$		
Normalised slope of wing lift, $\bar{a}_W$	$0.7436 < \bar{a}_W < 0.889$		
Normalised moment of wing lift, $(\bar{a}_W * \lambda_W)$	$1.888 < (\bar{a}_W * \lambda_W) < 2.033$		
Normalised moment arm for wing lift, λ_W	$2.125 < \lambda_W < 2.735$		

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